# Concurrency Theory

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# 4 hour course on concurrency theory

- Modelling Distributed systems
  - Labelled Transition Systems (Monday)
  - Petri nets (Thursday, if time allows, on whiteboard)
  - Process algebra – CCSP (Monday)

    (Denotational semantics on blackboard)
- When does an implementation meet a specification?
  - Semantic equivalences and preorders

    (Monday, after the break, on whiteboard)
- Does a system have a property?
  - Safety and liveness properties (Monday & Tuesday)
  - Temporal logic (Tuesday)
  - Progress, Justness and Fairness assumptions (Tuesday & Thursday)
- Session types (Thursday)
- Mutual exclusion (Thursday)

30-hour course on this subject, with recorded lectures:
`https://cgi.cse.unsw.edu.au/~rvg/6752`

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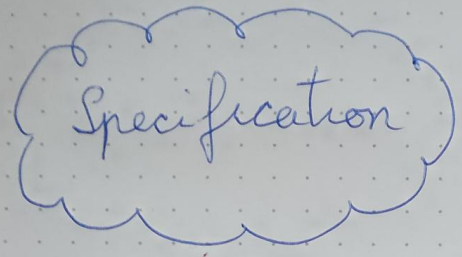
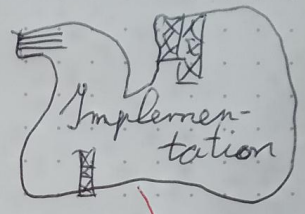
www.uni-trier.de




what we want

What we made

Object
or
Expression
in
mathematical Model
or
Specification Language

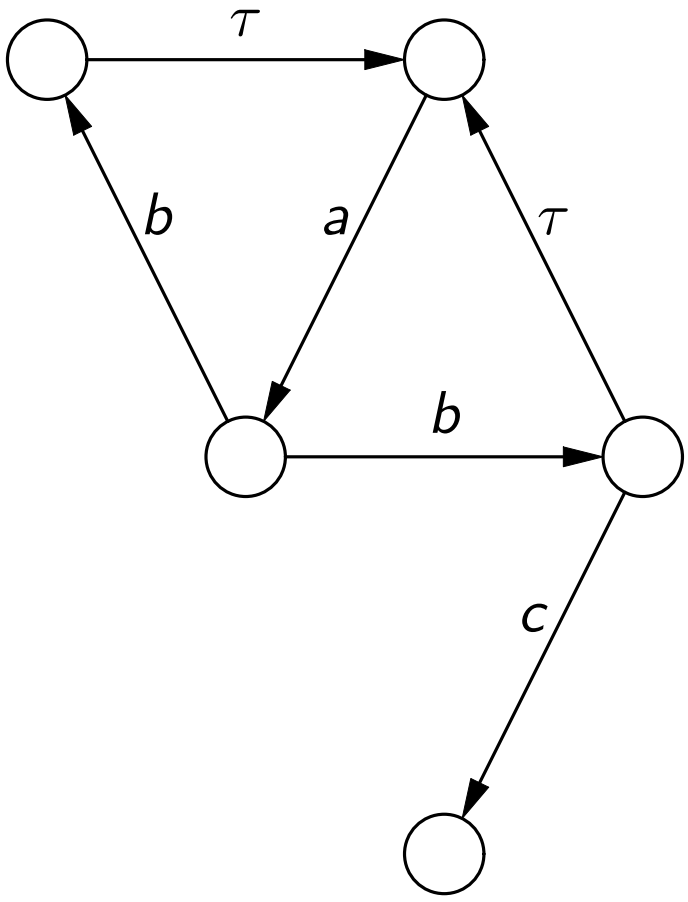
?
=

Object
or
Expression
in
math. Model
or
Language

Verification

Semantic Equivalence or preorder

# Labelled Transition Systems




A *labelled transition system*

| | |
|---|---|
| $\tau$ | internal action |
| $a$ | insert coin in slot |
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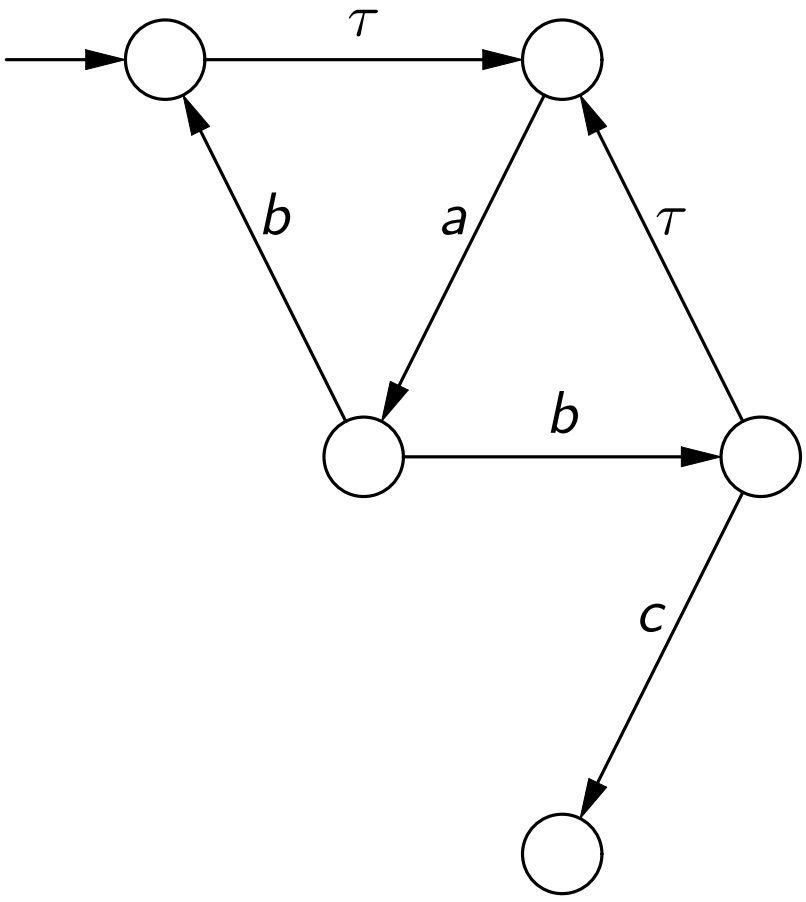
# Labelled Transition Systems




A *process graph*

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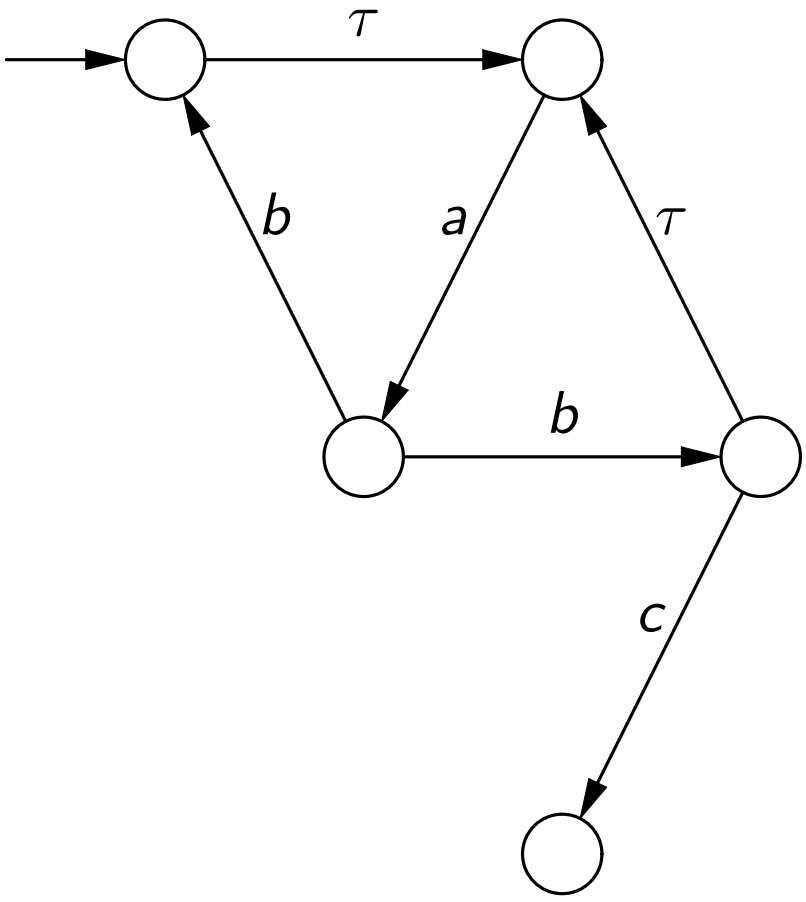


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(completed) traces:
$(ab)^\infty \cup ab(ab)^*c$.

# Labelled Transition Systems

Let $Act$ be a set of *actions*.

A *Labelled Transition System* over $Act$ is tuple $(S, \rightarrow)$ with

- $S$ a set (of *states*), and
- $\rightarrow \subseteq S \times Act \times S$, the *transition relation*.

# Process graphs

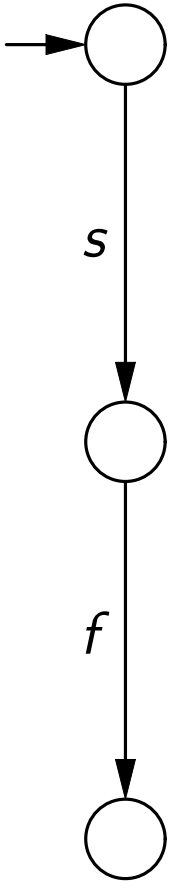
Let $Act$ be a set of *actions*.

A *Process graph* over $Act$ is tuple $(S, \rightarrow, I)$ with

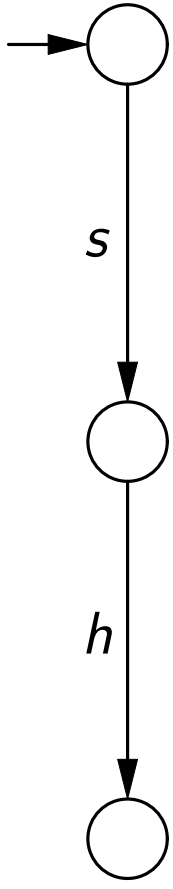
- $S$ a set (of *states*)
- $\rightarrow \subseteq S \times Act \times S$, the *transition relation*, and
- $I \in S$ the initial state.

# Relay Race

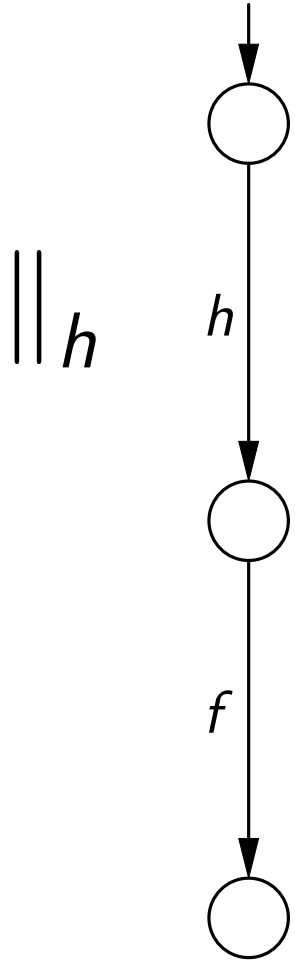
*Specification*




$s = start$
$h = handover$
$f = finish$

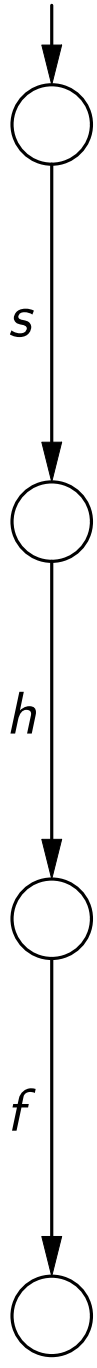



*Implementatation*




$\|_h$

$=$



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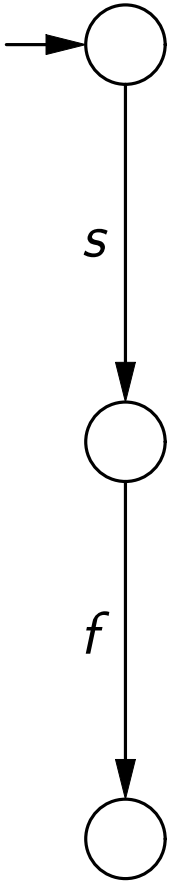


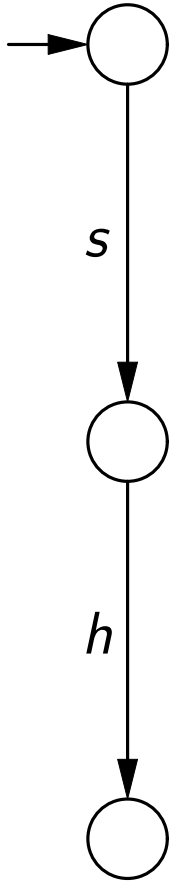


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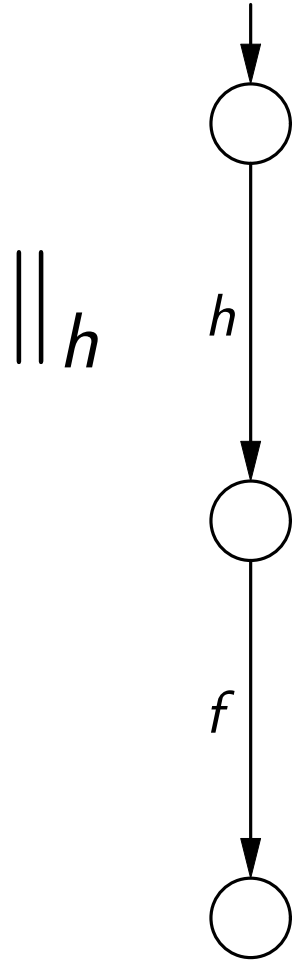


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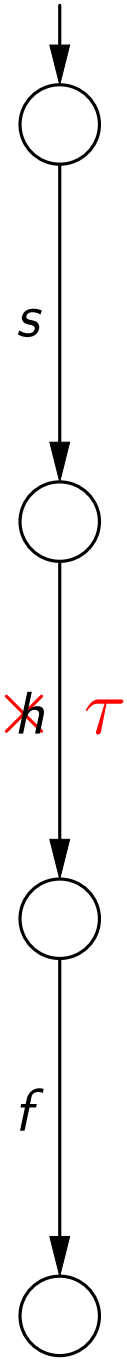


Abstraction

# The process algebra CCSP

$$P, Q ::= \; 0 \;\Big|\; \alpha.P \;\Big|\; P + Q \;\Big|\; P \parallel_S Q \;\Big|\; \tau_I(P) \;\Big|\; \text{Relabel} \;\Big|\; \text{Recursion}$$

with $\alpha \in Act := A \uplus \{\tau\}$, $S, I \subseteq A$.

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$s.h.0$

$coin.(chocolate.0 + pretzel.0)$

$VM := coin.(chocolate.VM + pretzel.VM)$

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$\tau_{\{h\}}(s.h \parallel_{\{h\}} h.f)$

# Operational semantics of CCSP

$$\alpha.P \xrightarrow{\alpha} P \qquad \frac{P \xrightarrow{\alpha} P'}{P + Q \xrightarrow{\alpha} P'} \qquad \frac{Q \xrightarrow{\alpha} Q'}{P + Q \xrightarrow{\alpha} Q'} \qquad \frac{P \xrightarrow{\alpha} P'}{\mathcal{R}(P) \xrightarrow{\beta} \mathcal{R}(P')} \begin{pmatrix} \alpha=\beta=\tau \vee \\ (\alpha,\beta) \in \mathcal{R} \end{pmatrix}$$

$$\frac{P \xrightarrow{\alpha} P'}{P \parallel_S Q \xrightarrow{\alpha} P' \parallel_S Q} (\alpha \notin S) \qquad \frac{P \xrightarrow{a} P' \quad Q \xrightarrow{a} Q'}{P \parallel_S Q \xrightarrow{a} P' \parallel_S Q'} (a \in S) \qquad \frac{Q \xrightarrow{\alpha} Q'}{P \parallel_S Q \xrightarrow{\alpha} P \parallel_S Q'} (\alpha \notin S)$$

$$\frac{P \xrightarrow{\alpha} P'}{\tau_I(P) \xrightarrow{\alpha} \tau_I(P')} (\alpha \notin I) \qquad \frac{P \xrightarrow{a} P'}{\tau_I(P) \xrightarrow{\tau} \tau_I(P')} (a \in I) \qquad \frac{P \xrightarrow{\alpha} P'}{X \xrightarrow{\alpha} P'} (X := P)$$

# Homework

Consider a process with the following set of completed traces: $\{bad, abd, adb, acd, adc\}$. Thus, in each run, the actions a, d and one out of c or b occur once each, subject to the following restrictions:

- b and c can never occur in the same trace.
- if c happens, it must be after a, and
- a happens before d.

Give a CCSP expression for this process that uses at most one $+$.

# Compositionality

$P_1 \| \cdots \| P_n$

$n$ parallel components of $k$ states each: $n^k$ states total.

*state explosion problem*

If we replace $P_i$ by an equivalent component $Q_i$, for $i = 1, \ldots, n$ then we want the new system $Q_1 \| \cdots \| Q_n$ to be equivalent to the old system $P_1 \| \cdots \| P_n$.

This is called *compositionality*.

Next to abstraction, this is the most important tool in combatting the state explosion problem.

# Safety and Liveness

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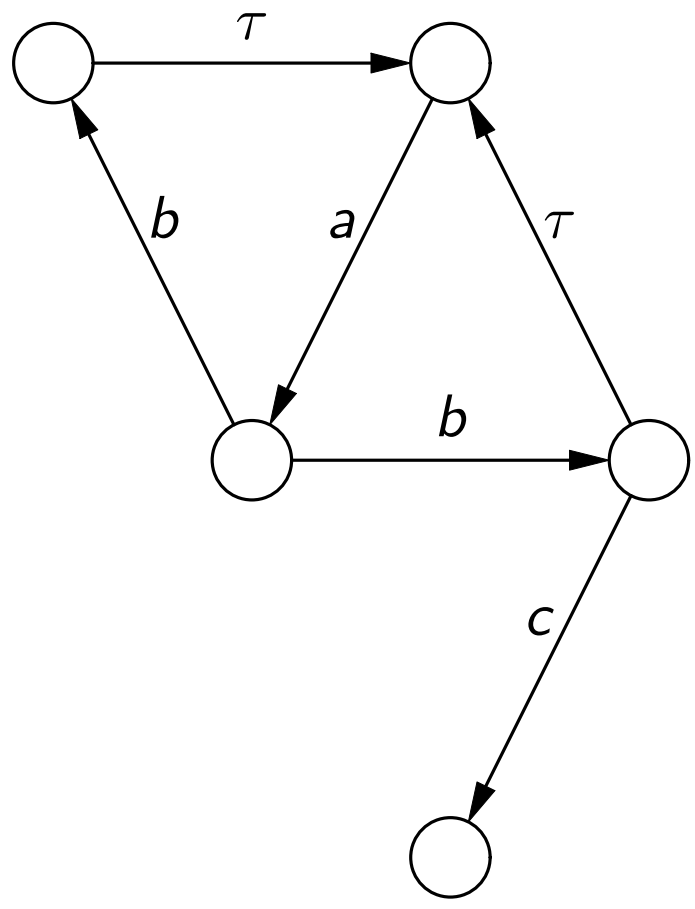


*Safety:*

*nothing bad will ever happen*

*Liveness:*

*something good will ever happen eventually*

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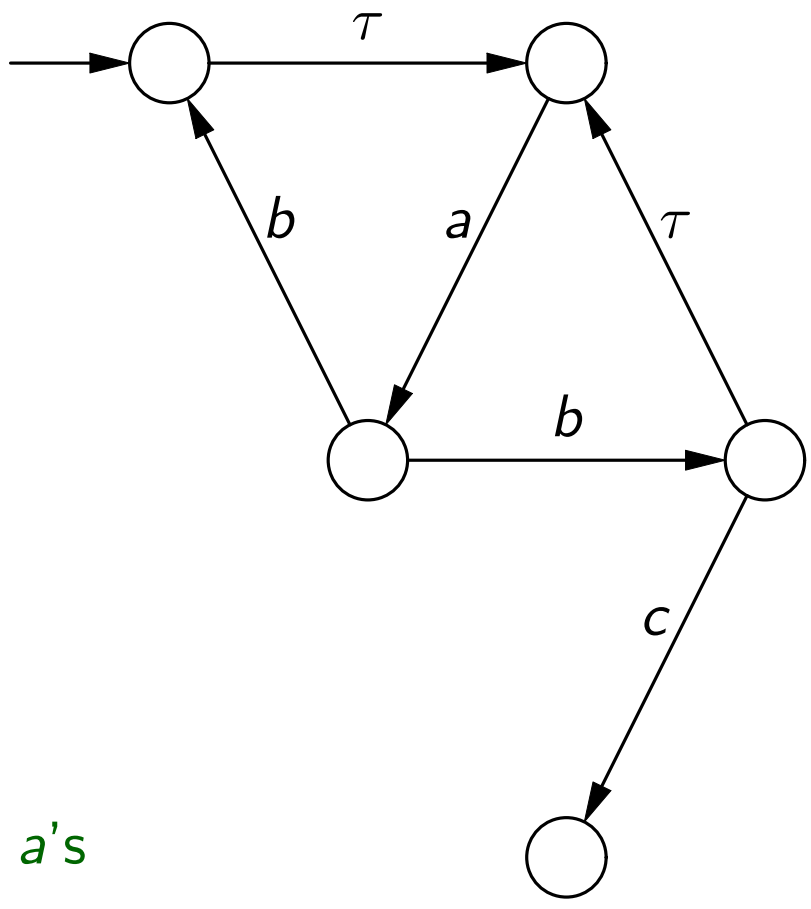


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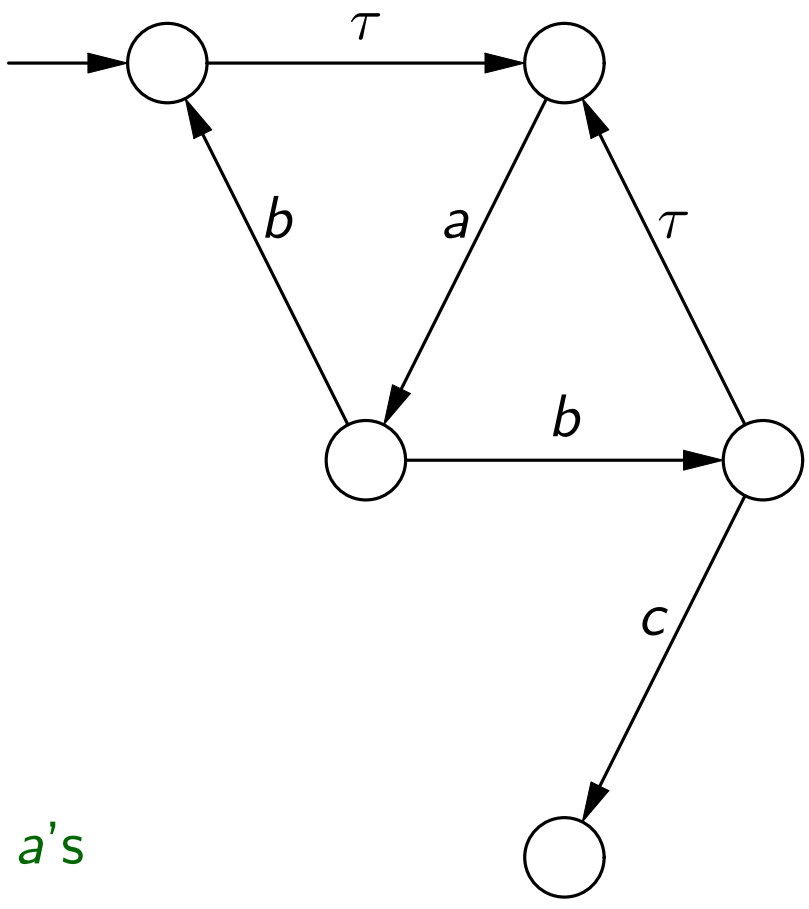


*Safety:*

*nothing bad will ever happen*

no run contains more $b$'s than $a$'s

*Liveness:*

*something good will ever happen eventually*

each $a$ will eventually be followed by a $b$

$\mathsf{VM} = \mathsf{a}.\mathsf{VM}$

# Pretzels

Consider a vending machine that alternatingly accepts a coin ($c$) and produces a pretzel ($p$).

$$VM = c.p.VM$$

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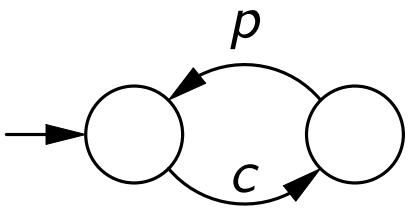


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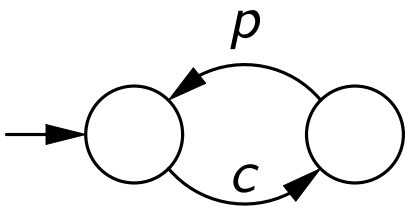


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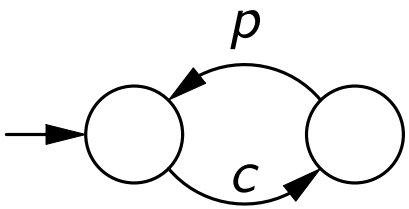


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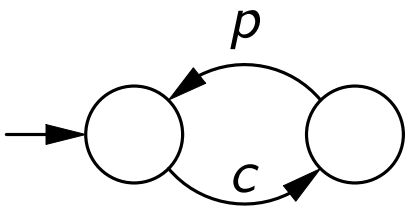


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Temporal Logic is a formalism for specifying (desired) properties of systems.

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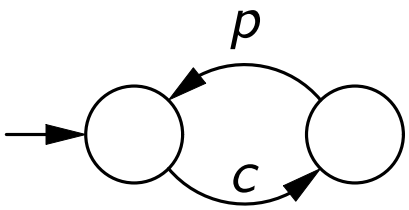


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Temporal Logic is a formalism for specifying (desired) properties of systems. It can tell what should happen, and in which order, but not when.

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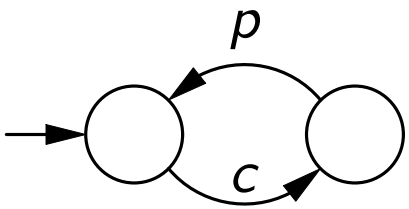


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Linear-time Temporal Logic (LTL) is a formalism for specifying (desired) properties of systems. It can tell what should happen, and in which order, but not when. [Pnueli 1977]

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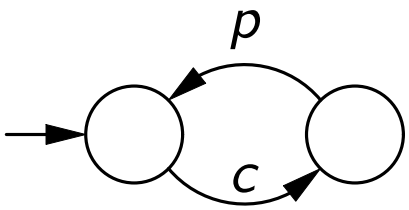


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In LTL we have $(\mathbf{c} \Rightarrow \mathbf{p})$. ← formula

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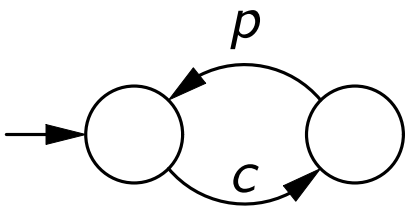


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In LTL we have $(\mathbf{c} \Rightarrow \mathbf{Fp})$. ← a temporal formula

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**F** (or $\diamond$) means eventually

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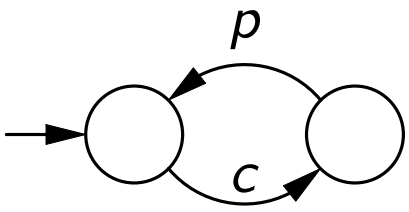


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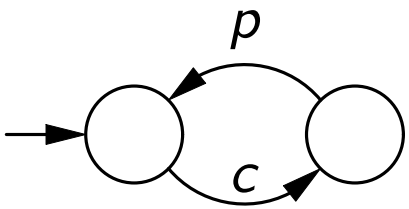


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In LTL we have $VM \models \mathbf{G}(\mathbf{c} \Rightarrow \mathbf{F}\mathbf{p})$. $\leftarrow$ a temporal judgement

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$\models$ means satisfies.

# Kripke structures

Let $AP$ be a set of *atomic predicates*.

A *Kripke structure* over $AP$ is tuple $(S, \rightarrow, \models)$ with

- $S$ a set (of *states*),
- $\rightarrow \subseteq S \times S$, the *transition relation*, and
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$s \models p$ says that predicate $p \in AP$ *holds* in state $s \in S$.

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A *completeness criterion* defines the *complete paths* as a subset of all paths. Complete paths model actual runs of the represented system.

The default completeness criterion, called progress, classifies a path as complete iff it either is infinite or ends in a deadlock state.

# LTL

*Linear-time temporal logic* (LTL) is designed to formulate useful properties of systems. Its syntax is

$$\varphi, \psi ::= p \mid \neg\varphi \mid \varphi \wedge \psi \mid \mathbf{X}\varphi \mid \mathbf{F}\varphi \mid \mathbf{G}\varphi \mid \psi \mathbf{U} \varphi$$

with $p \in AP$ an atomic predicate. The propositional connectives $\Rightarrow$ and $\vee$ can be added as syntactic sugar. It is interpreted on paths.

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$\pi \models \varphi$ says that the path $\pi$ *satisfies* the formula $\varphi$, or that $\varphi$ is *valid* on $\pi$. This is inductively defined by

- $\pi \models p$, with $p \in AP$, iff $s \models p$, where $s$ is the first state of $\pi$,
- $\pi \models \neg\varphi$ iff $\pi \not\models \varphi$,
- $\pi \models \varphi \wedge \psi$ iff $\pi \models \varphi$ and $\pi \models \psi$,
- $\pi \models \mathbf{X}\varphi$ iff $|\pi| > 0$ and $\pi_{+1} \models \varphi$,
- $\pi \models \mathbf{F}\varphi$ iff $\pi{\restriction}\rho \models \varphi$ for some finite prefix $\rho$ of $\pi$,
- $\pi \models \mathbf{G}\varphi$ iff $\pi{\restriction}\rho \models \varphi$ for each finite prefix $\rho$ of $\pi$, and
- $\pi \models \psi\mathbf{U}\varphi$ iff $\pi{\restriction}\rho \models \varphi$ for some finite prefix $\rho$ of $\pi$, and $\pi{\restriction}\zeta \models \psi$ for each proper prefix $\zeta$ of $\rho$.

# Interpreting LTL on states

Let $CC$ be a completeness criteria.

For $s$ a state in a Kripke structure, write
$P \models^{CC} \varphi$ iff $\pi \models \varphi$ for all complete paths starting in $s$.

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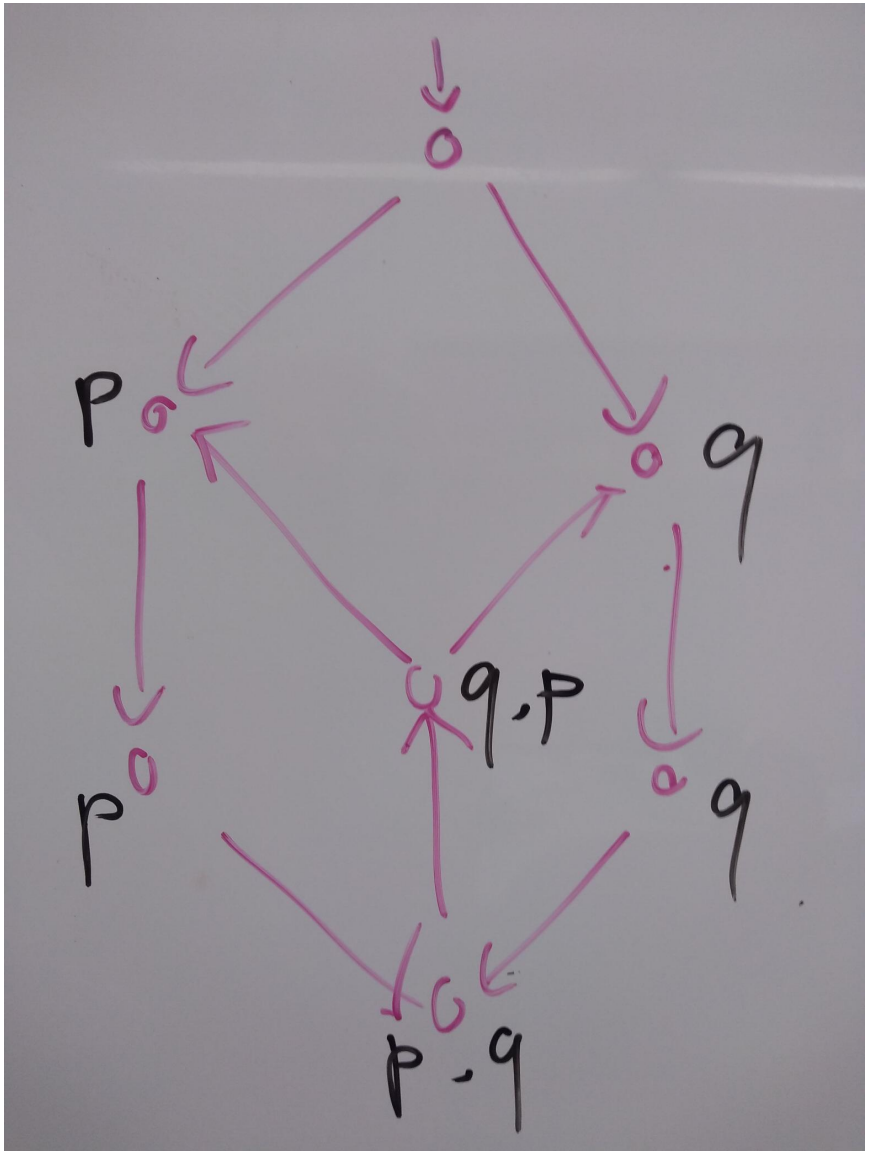
For $s$ a state in a Kripke structure, write
$P \models^{CC} \varphi$ iff $\pi \models \varphi$ for all complete paths starting in $s$.

We leave out the $CC$ and write $P \models \varphi$ when taking *progress* as $CC$.

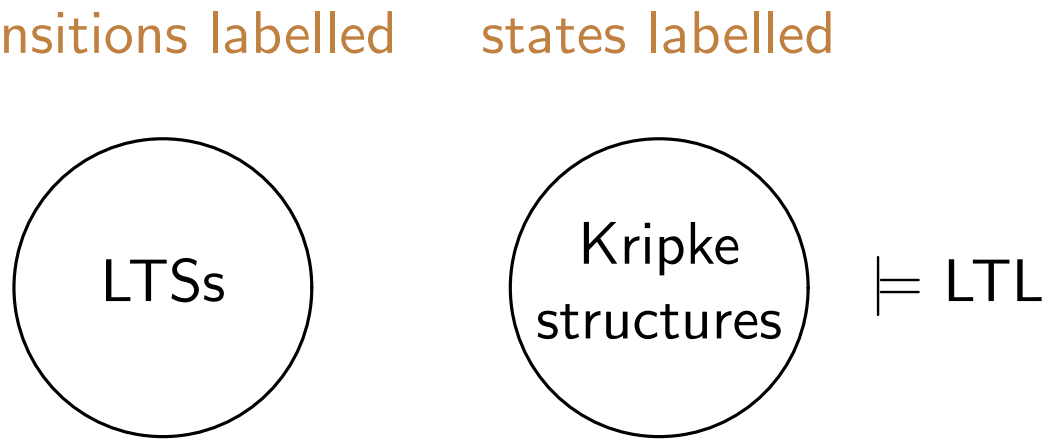
# Homework

Which of these LTL formulae hold in the initial state of this Kripke structure?

(a) $\mathbf{G}(p \vee q)$
(b) $\mathbf{FG}(p \vee q)$
(c) $\mathbf{F}(\mathbf{G}p \vee \mathbf{G}q)$
(d) $\mathbf{G}(q \rightarrow (q\mathbf{U}p))$
(e) $\mathbf{G}((p \wedge \neg q) \rightarrow \mathbf{X}q)$
(f) $\mathbf{G}((p \wedge q) \rightarrow \mathbf{X}(q\mathbf{U}p))$



# De Nicola - Vaandrager translation



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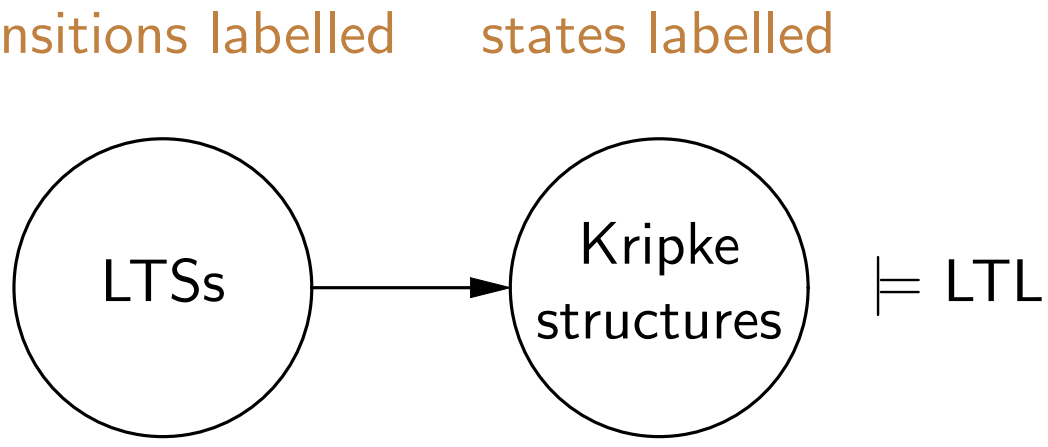

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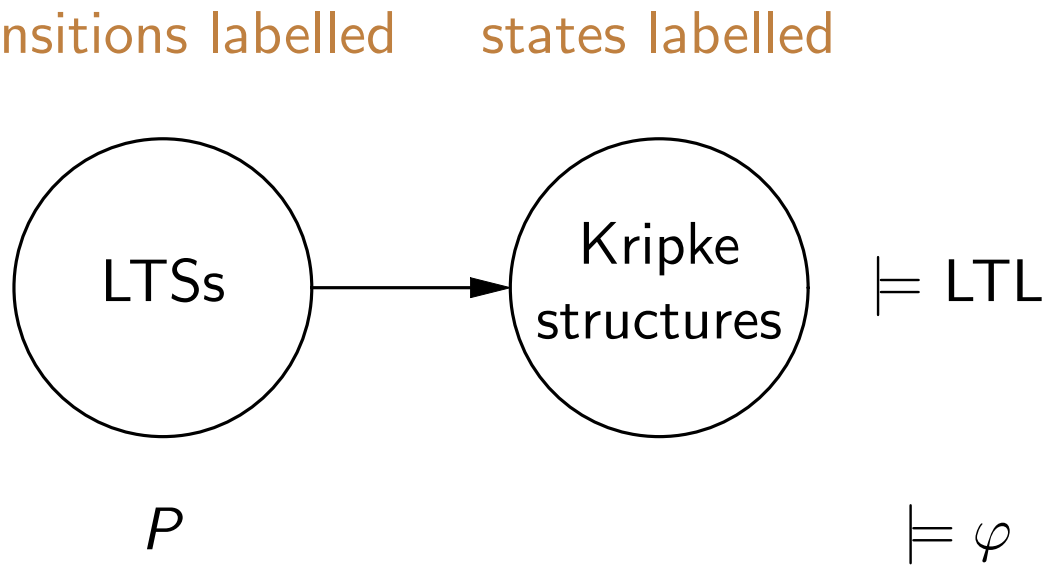
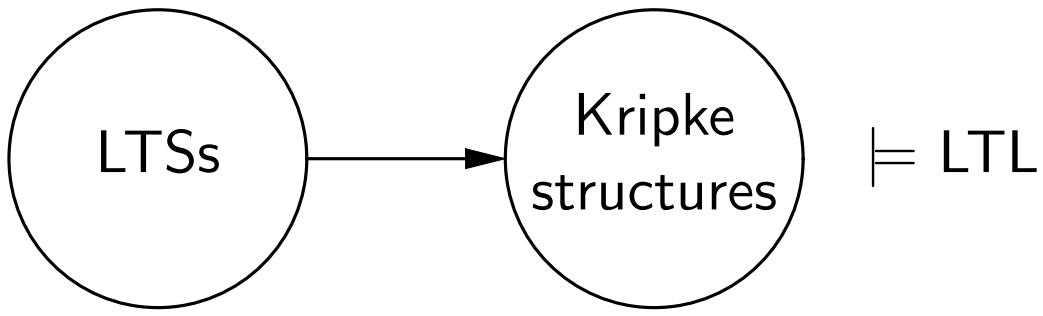


# De Nicola - Vaandrager translation

transitions labelled  states labelled




$P \quad \longmapsto \quad s \quad \models \varphi$

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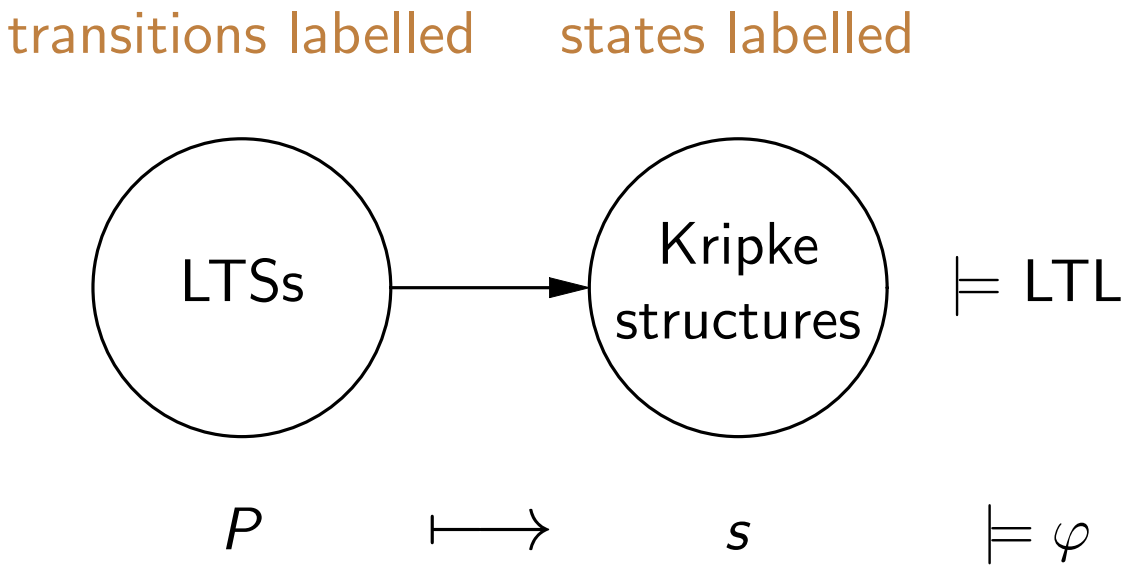


Create a new state halfway along any transition labelled by a visible action, and move the transition label to that state.

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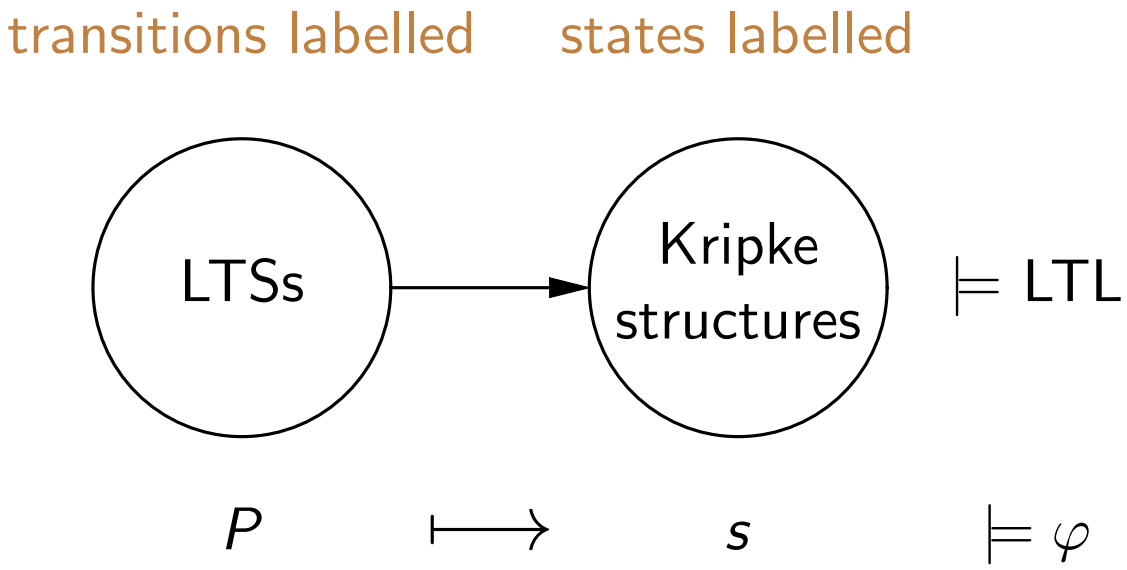


Create a new state halfway along any transition labelled by a visible action, and move the transition label to that state. $\tau$-transitions are not affected.

# Pretzels

Consider a vending machine that alternatingly accepts a coin ($c$) and produces a pretzel ($p$).

$$VM = c.p.VM$$

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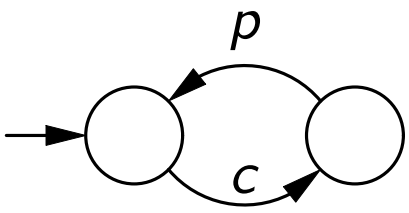


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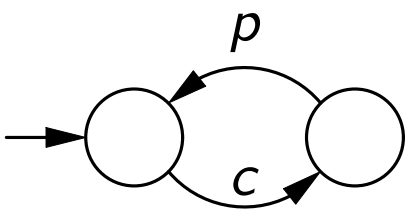


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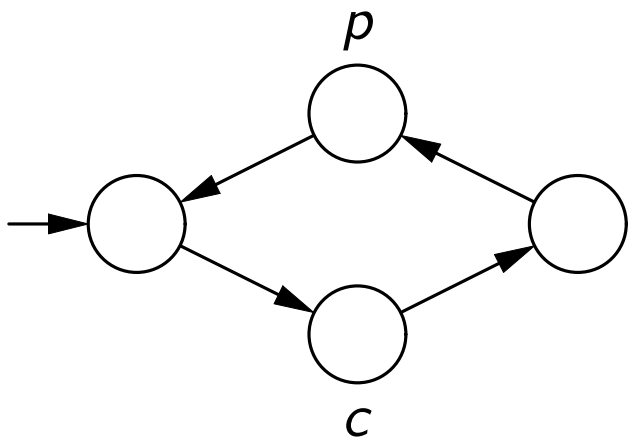

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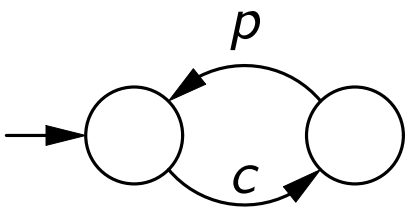


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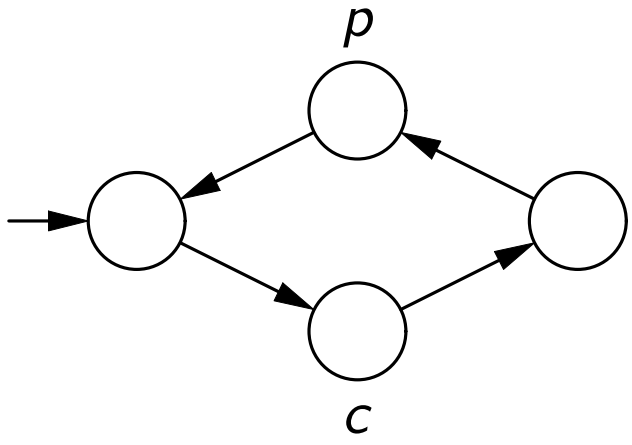


In LTL we have $VM \models \mathbf{G}(c \Rightarrow \mathbf{F}p)$.

*whenever a coin is inserted, eventually a pretzel is produced.*

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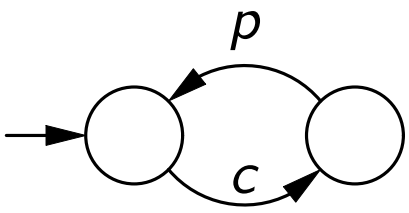


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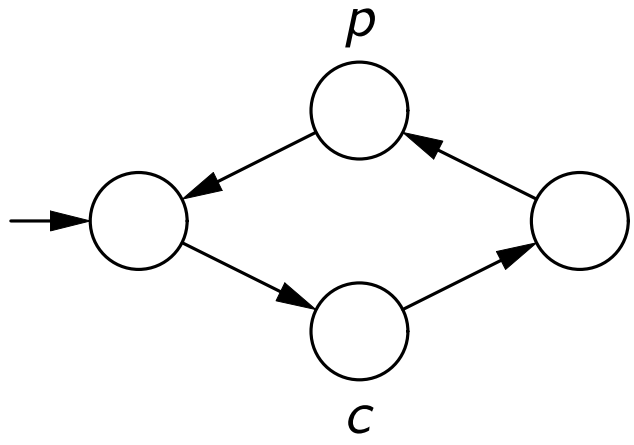


In LTL we have $VM \models \mathbf{G}(c \Rightarrow \mathbf{F}p)$.

*whenever a coin is inserted, eventually a pretzel is produced.*

But we also have $VM \models \mathbf{G}(p \Rightarrow \mathbf{F}c)$.

*whenever a pretzel is produced, eventually a new coin will be inserted.*

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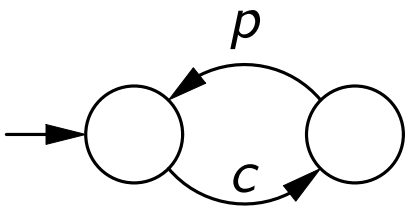


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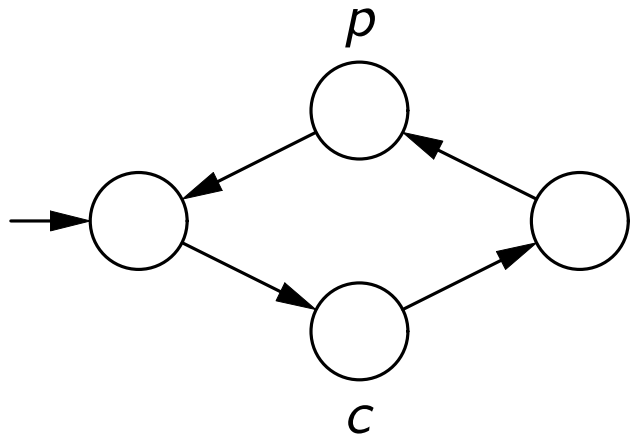


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But we also have $VM \models \mathbf{G}(p \Rightarrow \mathbf{F}c)$.

*whenever a pretzel is produced, eventually a new coin will be inserted.*

This example shows that standard LTL is not suitable to correctly describe the behaviour of this vending machine.

# Reactive Temporal Logic

Temporal judgements $P \models^{CC}_{B} \varphi$

$P$ is a process.

$\varphi$ is a temporal formula in LTL, CTL, or some other logic.

$B$ is a set of actions that may be blocked by the environment.

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$CC$ is a *completeness criterion*, such as progress, justness, weak fairness or strong fairness.

In default temporal logic infinite paths are complete; finite paths not.

$B$ declares some of the finite paths as complete.

$CC$ declares some of the infinite paths as incomplete.

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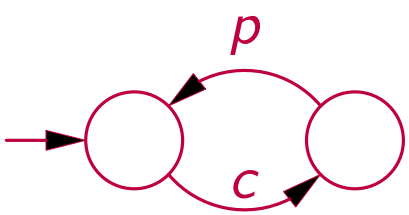


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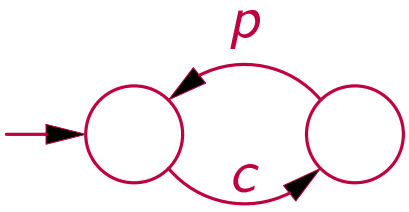


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In LTL we had $VM \models \mathbf{G}(c \Rightarrow \mathbf{F}p)$.

*whenever a coin is inserted, eventually a pretzel is produced.*

But $VM \models \mathbf{G}(p \Rightarrow \mathbf{F}c)$.

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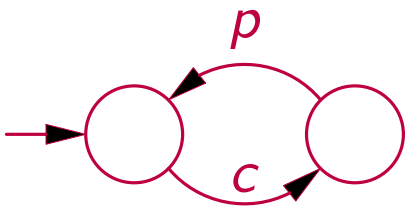


$$VM = c.p.VM$$
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In reactive LTL we have $VM \models_B \mathbf{G}(c \Rightarrow \mathbf{F}p)$.

*whenever a coin is inserted, eventually a pretzel is produced.*

But $VM \not\models_B \mathbf{G}(p \Rightarrow \mathbf{F}c)$.

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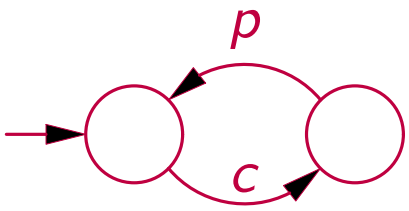


$$VM = c.p.VM$$
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In reactive LTL we have $VM \models_B \mathbf{G}(c \Rightarrow \mathbf{F}p)$.

*whenever a coin is inserted, eventually a pretzel is produced.*

But $VM \not\models_B \mathbf{G}(p \Rightarrow \mathbf{F}c)$. We still have $VM \models_\emptyset \mathbf{G}(p \Rightarrow \mathbf{F}c)$.

*whenever a pretzel is produced, eventually a new coin will be inserted.*

# Reactive LTL

LTL formula are traditionally interpreted on infinite paths.

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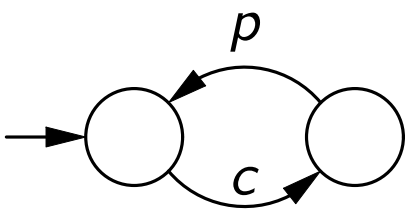


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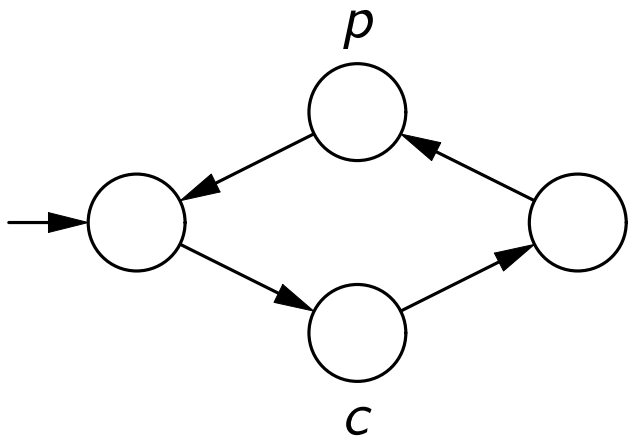


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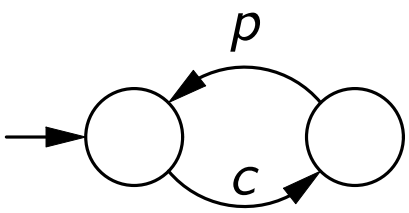


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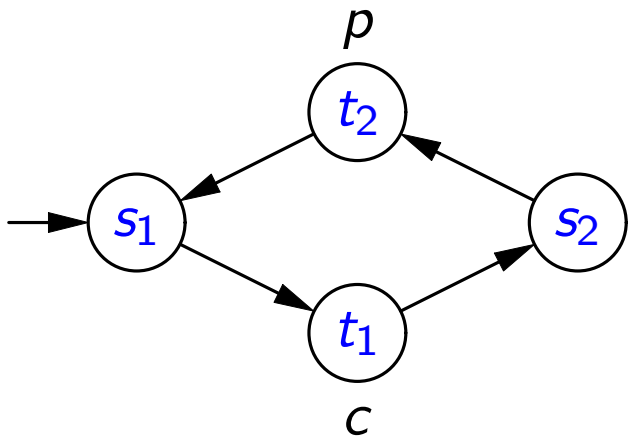


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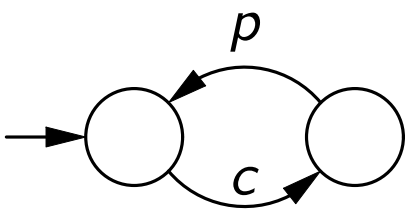


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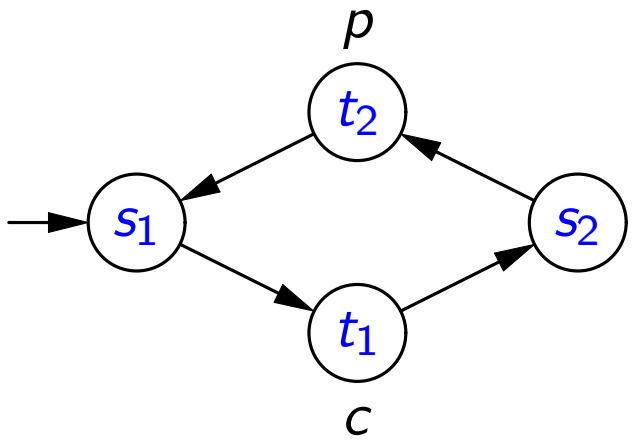


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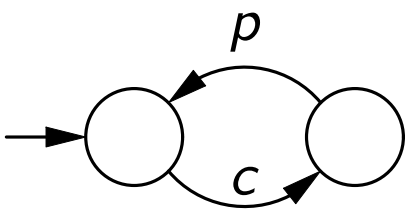


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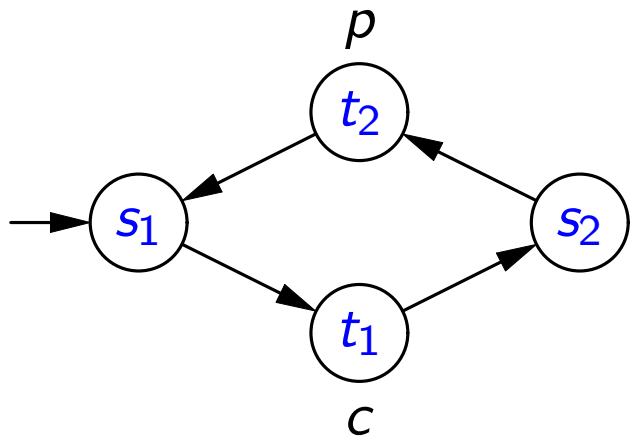


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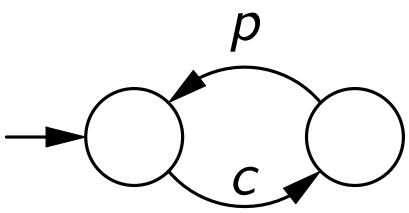


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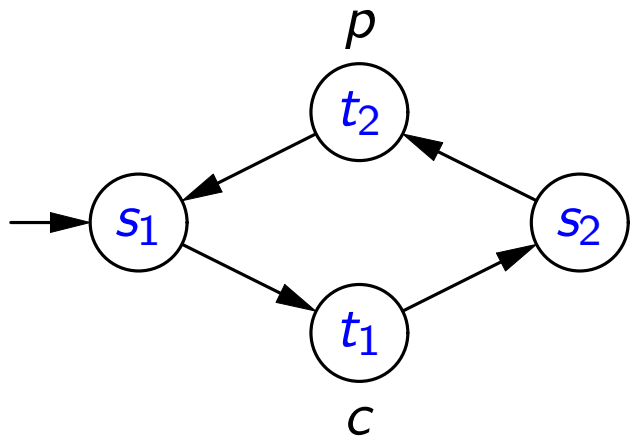


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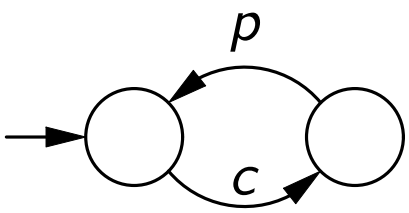


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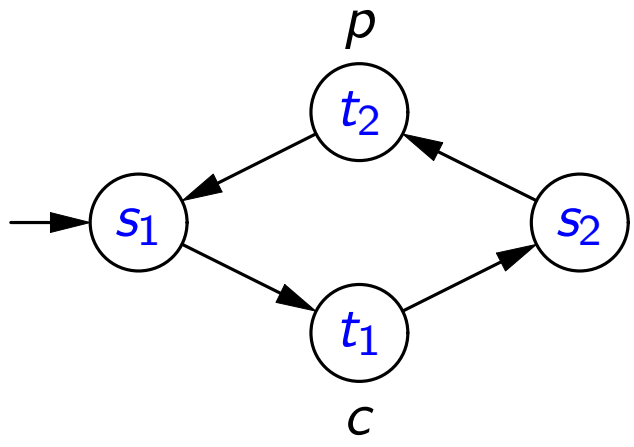


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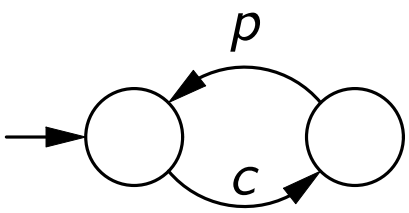
# Reactive LTL

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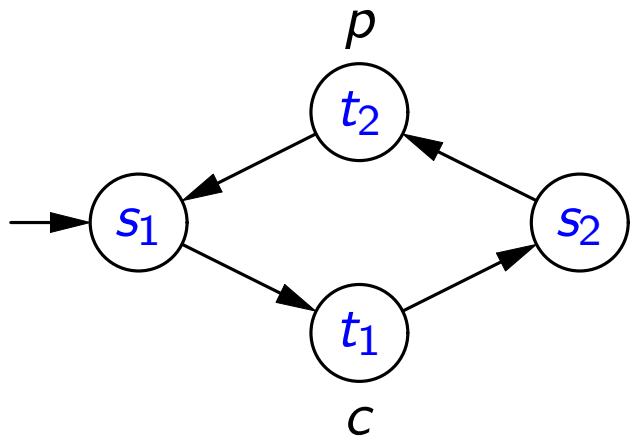


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$s_1 t_1 s_2 t_2 s_1 t_1 s_2 t_2 s_1 \cdots \models \mathbf{G}(p \Rightarrow \mathbf{F}c)$ complete

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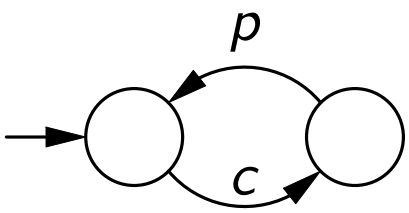
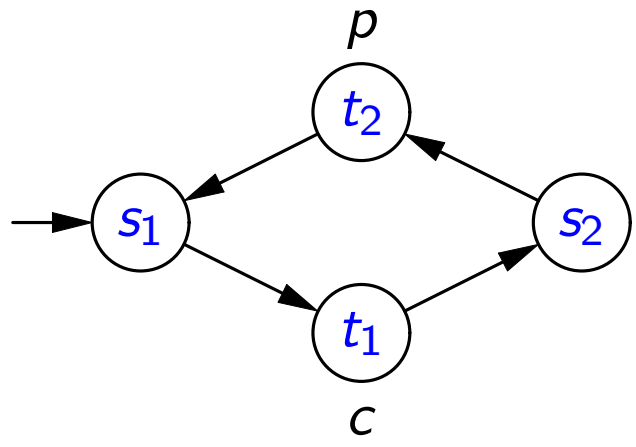
$s_1 t_1 s_2 t_2 s_1 t_1 s_2 \not\models \mathbf{G}(c \Rightarrow \mathbf{F}p)$ incomplete

# Reactive LTL




$VM = c.p.VM$

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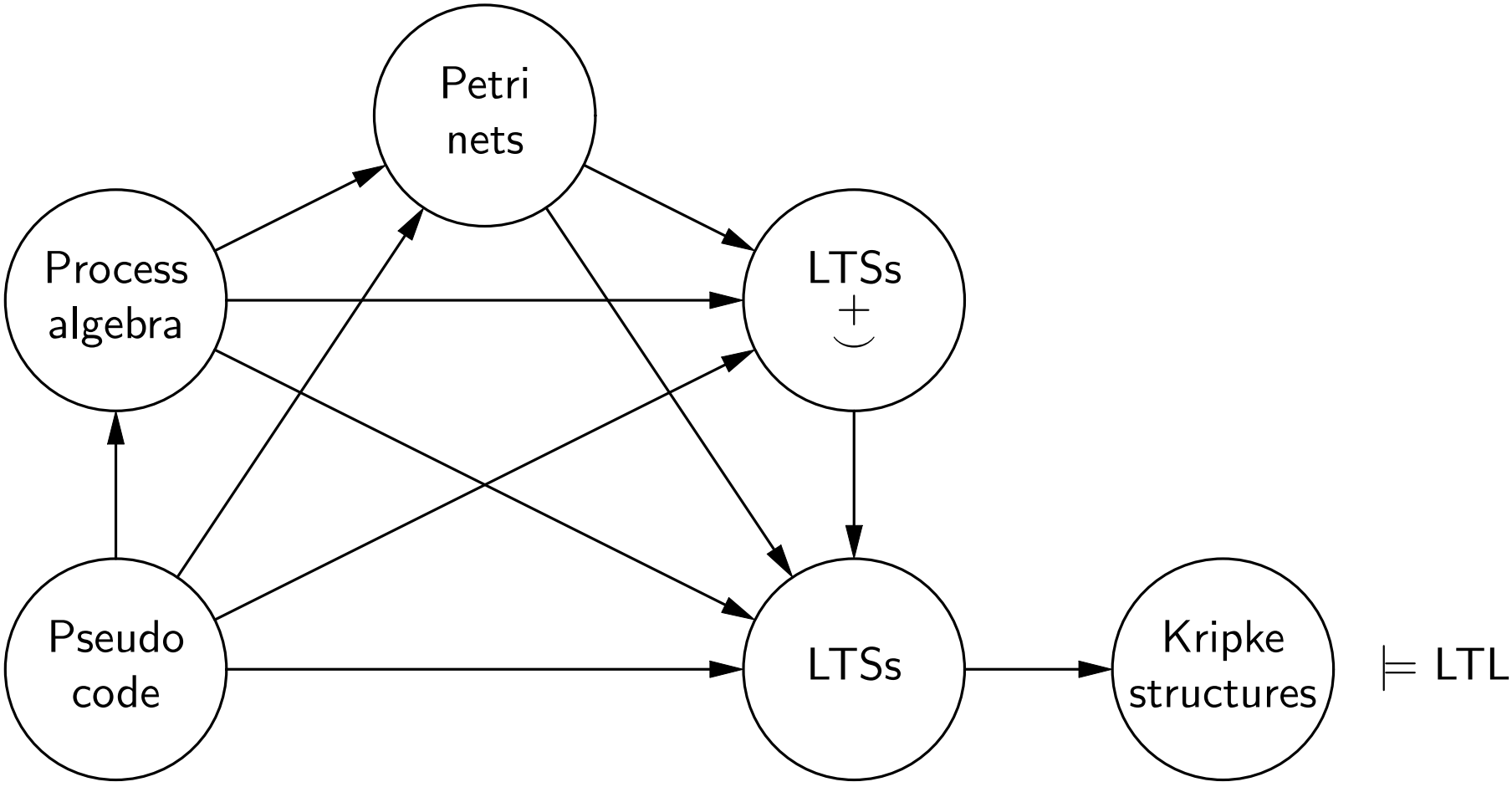
$$B = \{c\}$$

$P \models \varphi$ iff $\pi \models \varphi$ for all complete paths $\pi$.

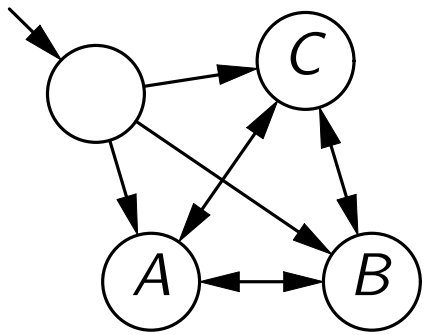
A path is complete iff either it is infinite, or its last state has outgoing transitions with labels from $B$ only.

# Models for distributed systems



# Fairness

Alice, Bart and Cameron stand behind a bar, ordering and drinking beer.
As there is only one barman, they are served sequentially.
None of them is served twice in a row, but each of them is ready
for the next beer as soon as another person is served.



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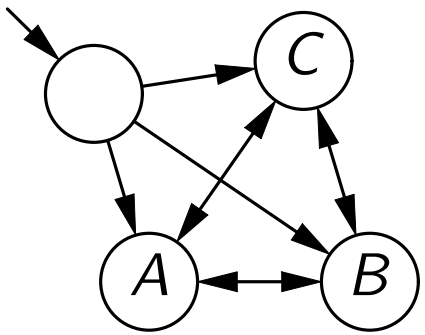


$\not\models \quad \mathbf{F}B$

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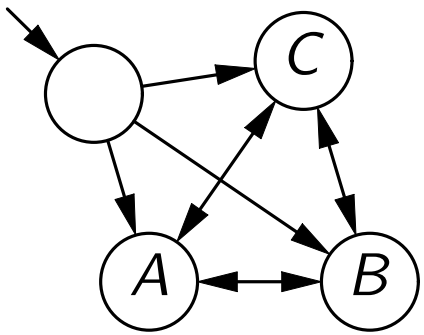


$\models^{WF} \mathbf{F}B$

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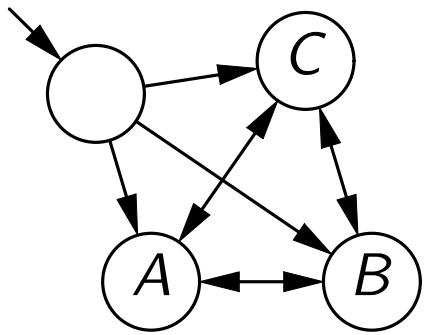
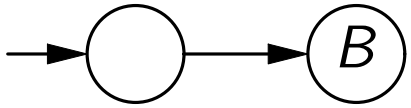



$\models^{WF} \mathbf{F}B$

Assuming fairness often leads to unwarranted conclusions.

# Progress

Bart is the only customer in a bar in London, with a single barman.
He only wants one beer.



# Progress

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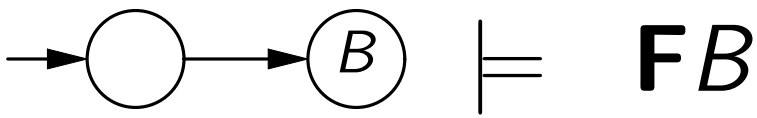
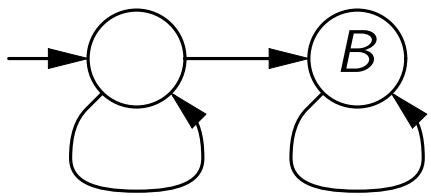


# Justness

Bart is the only customer in a bar in London, with a single barman.
He only wants one beer.
Alice and Cameron are in a bar in Tokyo. They drink a lot of beer.
There is no interaction of any kind between the two bars.
Yet, one may choose to model the drinking in these two bars as a single distributed system.



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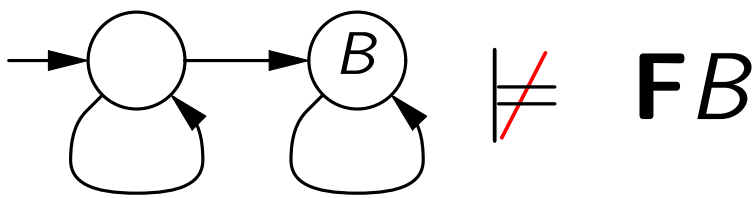

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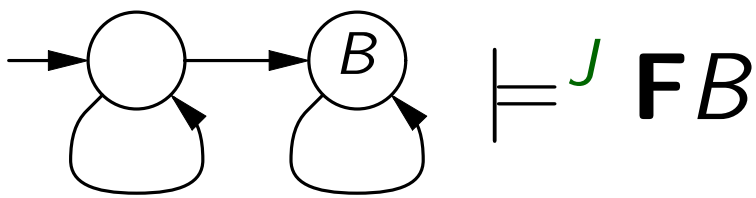


Each transition that is at some point enabled, will eventually occur, unless one of its necessary resources is used for another transition.

# Session types

Another process algebra:

$$
\begin{array}{rcl}
\mathrm{T} := & \textsc{ok} & \\
\mid & \bigoplus_{i \in I}\ p_i!\lambda_i; \mathrm{T}_i & \qquad \mathbb{N} := \quad p[\![\mathrm{T}]\!] \\
\mid & \sum_{i \in I}\ p_i?\lambda_i; \mathrm{T}_i & \qquad\quad \mid \quad 0 \\
\mid & X & \qquad\quad \mid \quad \mathbb{N} \parallel \mathbb{N} \\
\mid & \mu X.\mathrm{T} &
\end{array}
$$

Example:

$$
\begin{array}{rl}
 & \mathit{seller1}[\![\mu X.\mathit{buyer1}?; X]\!] \\
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Lock-freedom: Everyone wishing to trade eventually does so.

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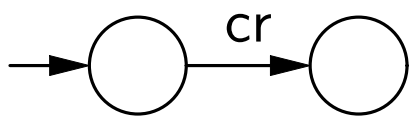
Lock-freedom: Everyone wishing to trade eventually does so.

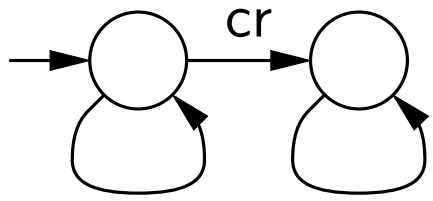
Theorem: $\mathcal{L}(J)$ holds iff network is well-typed and race-free.

# Justness

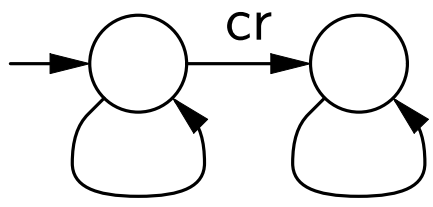
Cateline, eating a croissant in Paris:




In parallel with Alice, making an infinite sequence of phone calls in London:




Cateline, making an infinite sequence of choices between eating her croissant and making yet another phone call:




In Petri nets, these systems have totally different representations.

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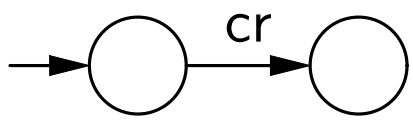


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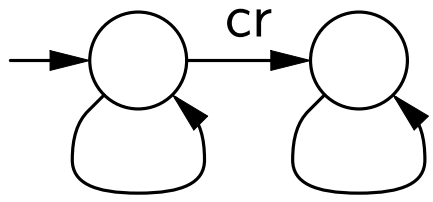


$\models^{J}_{\emptyset}$ $\mathbf{F}\,\mathrm{cr}$

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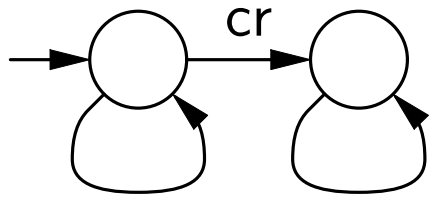


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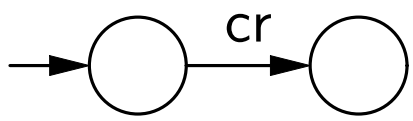


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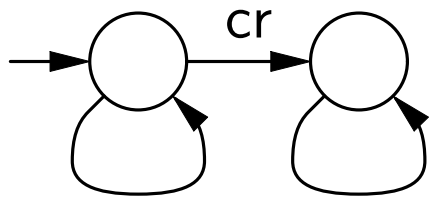


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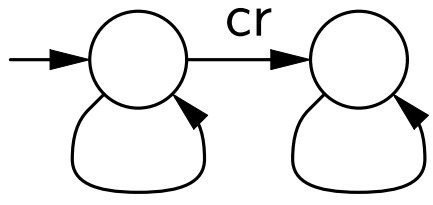


$\not\models^{J}_{\emptyset}$ **F** cr

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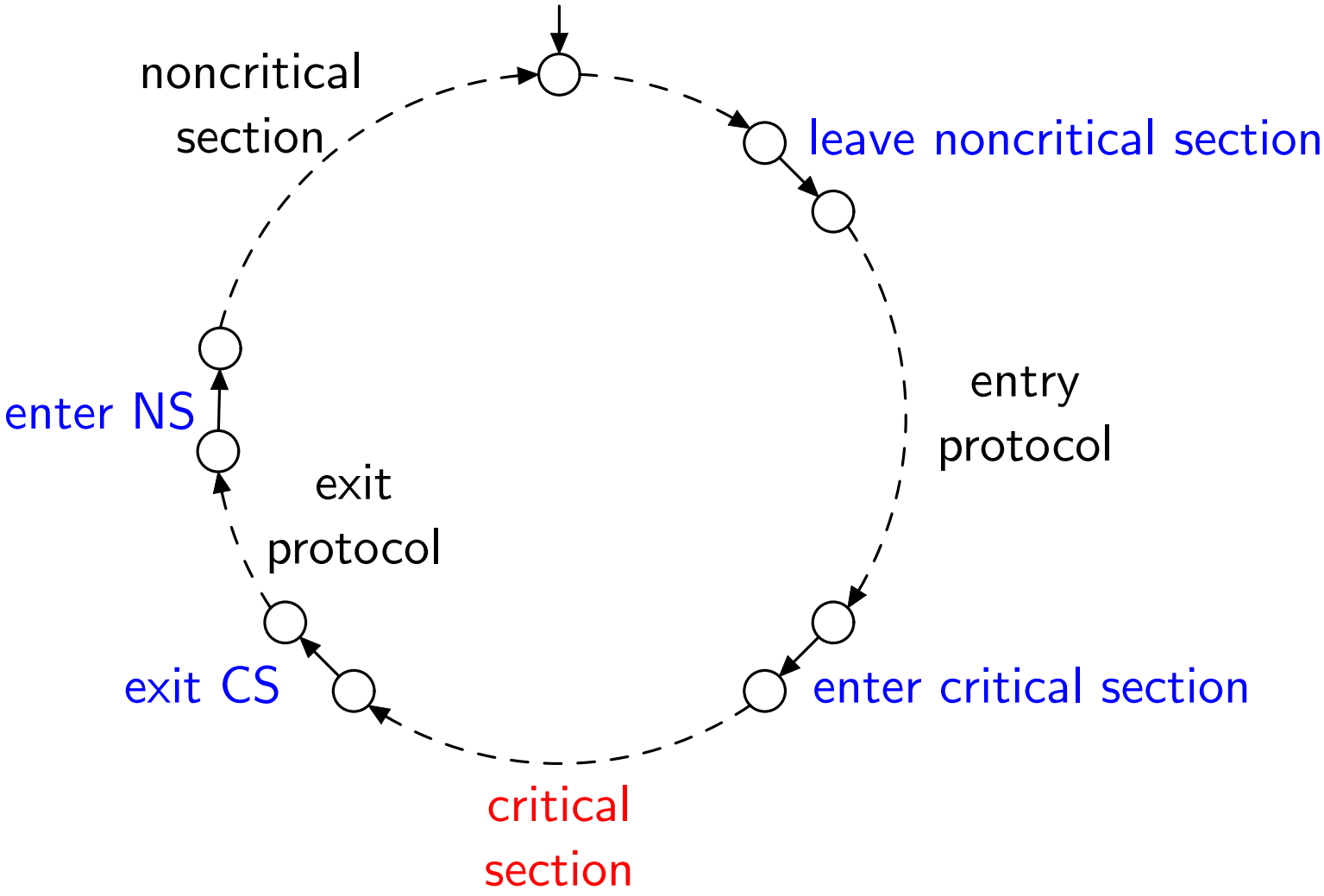
# Mutual exclusion



# Peterson's mutual exclusion algorithm

**Process A**

**repeat forever**

$\ell_1$ **leave noncritical section**
$\ell_2$ $readyA := true$
$\ell_3$ $turn := B$
$\ell_4$ **await** $(readyB = false \vee turn = A)$
$\ell_5$ **enter critical section**
$\ell_6$ **leave critical section**
$\ell_7$ $readyA := false$
$\ell_8$ **enter noncritical section**

**Process B**

**repeat forever**

$m_1$ **leave noncritical section**
$m_2$ $readyB := true$
$m_3$ $turn := A$
$m_4$ **await** $(readyA = false \vee turn = B)$
$m_5$ **enter critical section**
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*Peterson's algorithm (pseudocode)*

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Mutual exclusion: Never are both processes together in the critical section.
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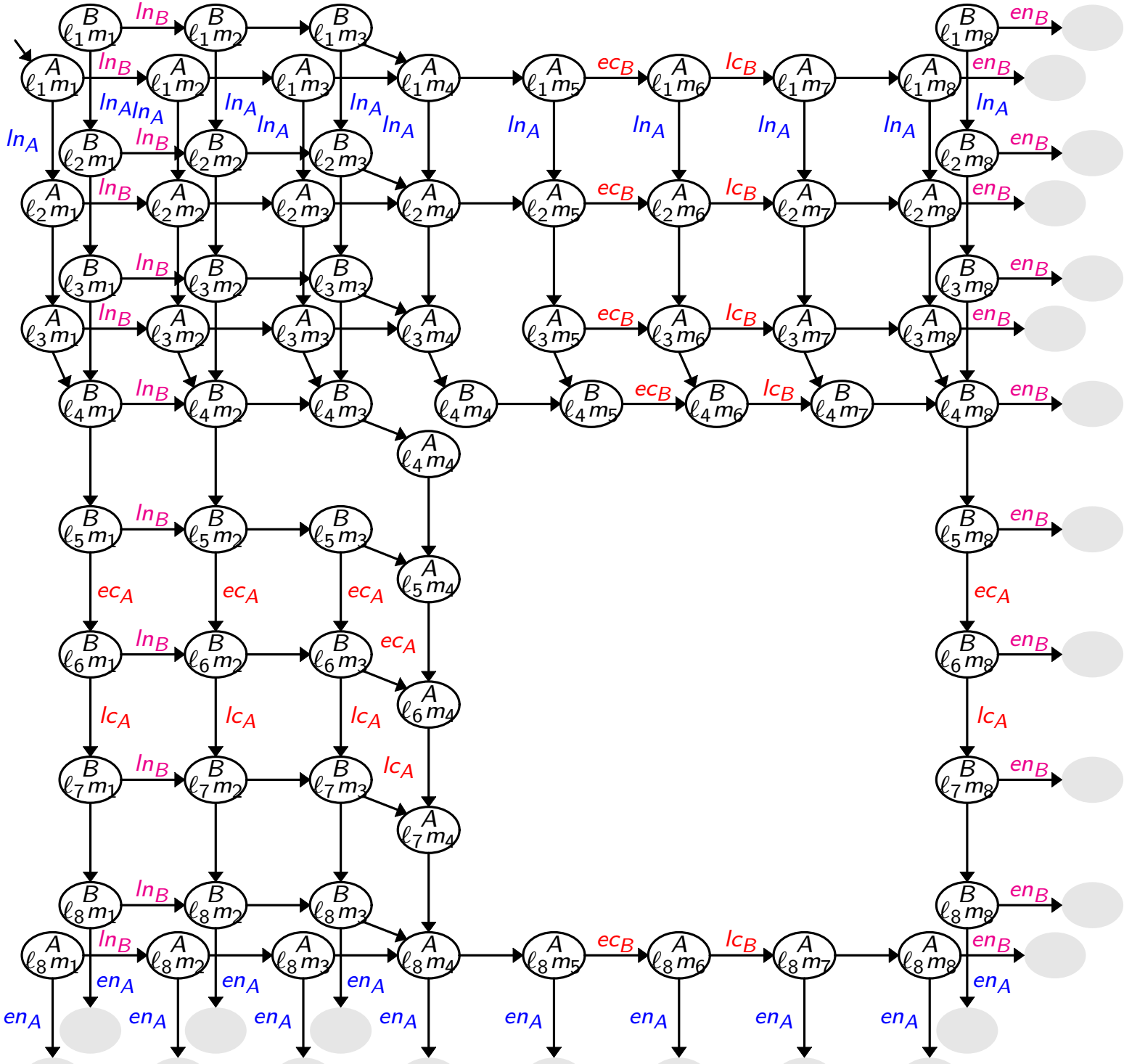
Speed independence: the protocol should work regardless how long things take.
Atomicity: a memory cell cannot handle a read and a write at the same time. So a write will have to wait until a pending read is finished.


$B\ \ell_1 m_1$
$ln_B$
$B\ \ell_1 m_2$
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$B\ \ell_1 m_8$
$en_B$
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$A\ \ell_1 m_2$
$A\ \ell_1 m_3$
$A\ \ell_1 m_4$
$A\ \ell_1 m_5$
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$A\ \ell_8 m_5$
$A\ \ell_8 m_6$
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$en_A$

# Correctness properties of MUTEX protocols

$$\text{(ORD)} \quad P \models \big((\neg\, act_i)\mathbf{W}\, ln_i\big) \wedge \mathbf{G}\,\big(ln_i \Rightarrow \mathbf{Y}\big((\neg\, act_i)\mathbf{W}\, ec_i\big)\big) \wedge \mathbf{G}\,\big(ec_i \Rightarrow \mathbf{Y}\big((\neg\, act_i)\mathbf{W}\, lc_i\big)\big) \wedge \mathbf{G}\,\big(lc_i \Rightarrow \mathbf{Y}\big((\neg\, act_i)\mathbf{W}\, en_i\big)\big) \wedge \mathbf{G}\,\big(en_i \Rightarrow \mathbf{Y}\big((\neg\, act_i)\mathbf{W}\, ln_i\big)\big)$$

for $i = 1, \ldots, N$. Here $act_i := (ln_i \vee ec_i \vee lc_i \vee en_i)$.

This requirement says that in a mutex protocol the actions $ln_i$, $ec_i$, $lc_i$ and $en_i$ must occur in the right order.

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for $i = 1, \ldots, N$. Here $act_i := (ln_i \vee ec_i \vee lc_i \vee en_i)$.

$$\text{(ME)} \quad P \models \mathbf{G}\,\big(ec_i \Rightarrow \big((\neg ec_j)\mathbf{W} lc_i\big)\big)$$

for all $i, j = 1, \ldots, N$ with $i \neq j$.

# Correctness properties of MUTEX protocols

$(\mathrm{EC}^J)$ $\quad P \models_B^J \mathbf{G}(\mathit{ln}_i \Rightarrow \mathbf{F}\mathit{ec}_i)$

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# Correctness properties of MUTEX protocols

$(\mathrm{EC}^J)$ $\quad P \models^J_B \mathbf{G}(ln_i \Rightarrow \mathbf{F}\, ec_i)$

for $i = 1, \ldots, N$.

$(\mathrm{LC}^J)$ $\quad P \models^J_B \mathbf{G}(ec_i \Rightarrow \mathbf{F}\, lc_i)$

$(\mathrm{EN}^J)$ $\quad P \models^J_B \mathbf{G}(lc_i \Rightarrow \mathbf{F}\, en_i)$

$(\mathrm{LN}^J)$ $\quad P \models^J_{B \setminus \{ln_i\}} \mathbf{F}\, ln_i \wedge \mathbf{G}(en_i \Rightarrow \mathbf{F}\, ln_i)$

for $i = 1, \ldots, N$.

# Memory model

The processes in a mutex protocol communicate with each other by writing to and reading from shared registers.

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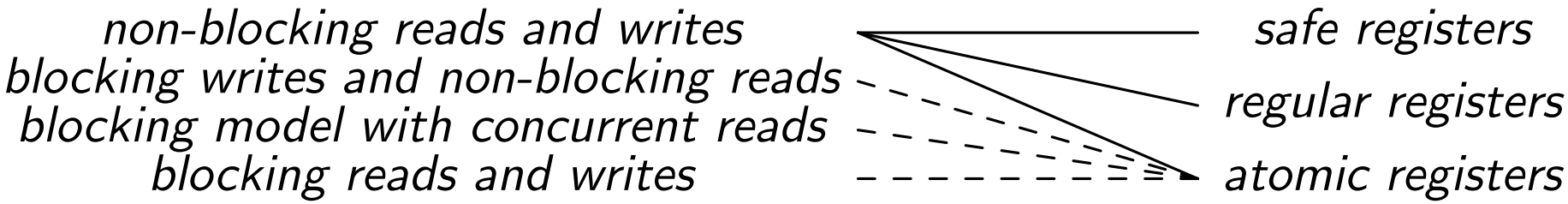

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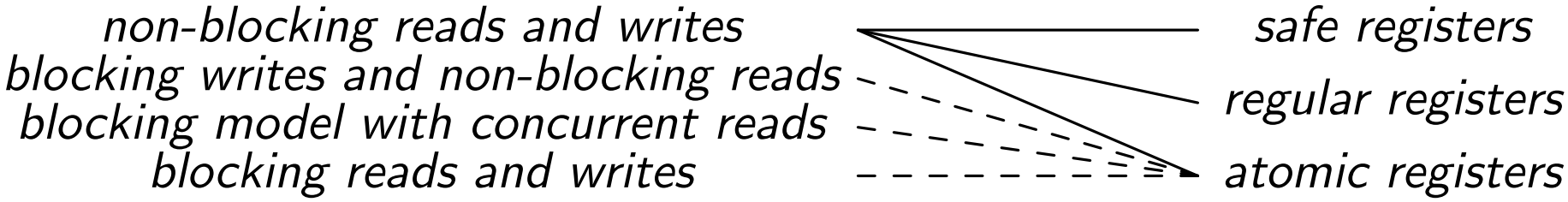


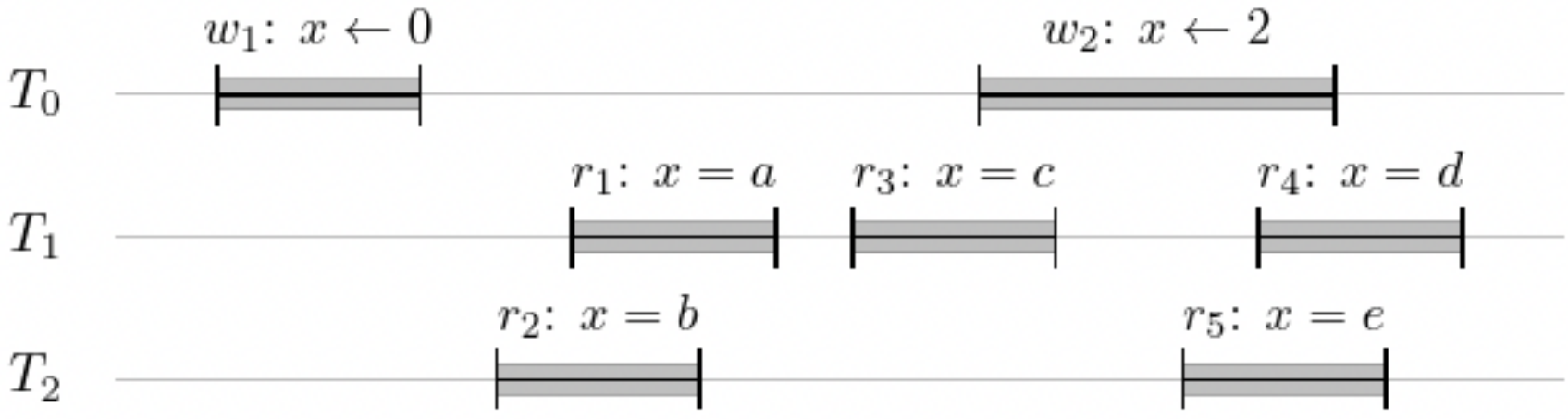

# Model checking

Traditional verification of mutual exclusion algorithms:
pen-and-paper proofs using behavioural reasoning.

> *"the behavioral reasoning used in our correctness proofs, and in most other published correctness proofs of concurrent algorithms, is inherently unreliable" [Lamport 1986]*

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But only finite state spaces.
So no bakery protocol
and we can check only for a small number of threads.

# Results

| *Algorithm* | *# threads* | *Safe* | *Regular* | | *Atomic* | | |
|---|---|---|---|---|---|---|---|
| | | *T* | *T* | *T* | *S* | *I* | *A* |
| Anderson | 2 | S | S | S | S | M | M |
| Aravind BLRU | 3 | S | S | S | M | M | M |
| Aravind BLRU (alt.) | 3 | S | S | S | S | M | M |
| Attiya-Welch (orig.) | 2 | D | S | S | D | M | M |
| Attiya-Welch (orig., alt.) | 2 | S | S | S | D | M | M |
| Attiya-Welch (var.) | 2 | M | M | S | D | M | M |
| Attiya-Welch (var., alt.) | 2 | S | S | S | D | M | M |
| Burns-Lynch | 3 | D | D | D | D | M | M |
| Dekker | 2 | M | M | S | D | M | M |
| Dekker (alt.) | 2 | M | M | S | S | M | M |
| Dekker RW-safe | 2 | S | S | S | D | M | M |
| Dekker RW-safe (DFtoSF) | 2 | S | S | S | S | M | M |
| Dijkstra | 3 | M | D | D | M | M | M |

# Results (continued)

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| Kessels | 2 | X | X | S | S | M | M |
| Knuth | 3 | M | S | S | M | M | M |
| Lamport 1-bit | 3 | D | D | D | D | M | M |
| Lamport 1-bit (DFtoSF) | 3 | S | S | S | S | M | M |
| Lamport 3-bit | 3 | S | S | S | S | M | M |
| Peterson | 2 | X | X | S | S | M | M |
| Szymanski flag (int.) | 3 | X | X | S | S | M | M |
| Szymanski flag (bit) | 3 | X | X | X | X | X | X |
| Szymanski 3-bit lin. wait | 3 | X | X | X | X | X | X |
| Szymanski 3-bit lin. wait (alt.) | 2 | S | S | S | S | M | M |

# Speed independence

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If two processes $A$ and $B$ are engaged in a race, and
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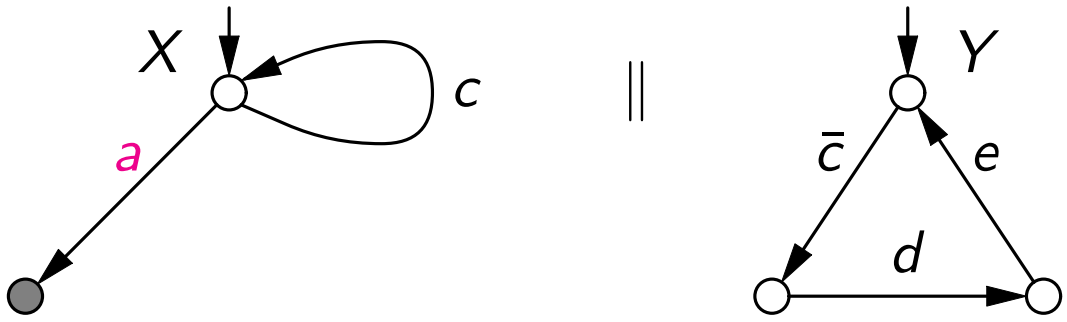
CCS process $(X|Y)\backslash c$ with $X \stackrel{def}{=} a.\mathbf{0} + c.X$ and $Y \stackrel{def}{=} \bar{c}.d.e.Y$.

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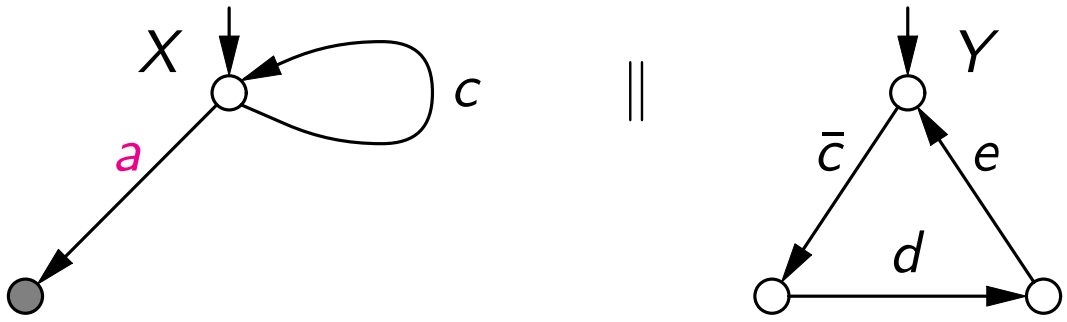


Here it is possible that $a$ never happens.

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# Impossibility of speed ind. mutual excl. under atomicity

For mutual exclusion to be possible, there must be a variable *readyP* that is written by Proc. $P$ to request entry to CS.

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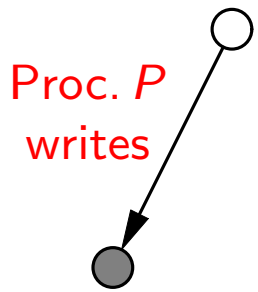


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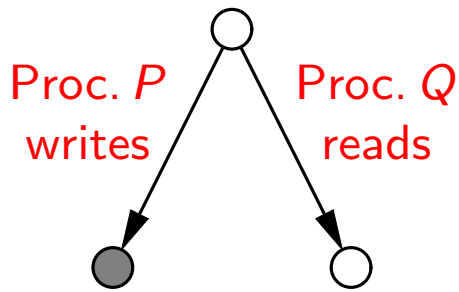


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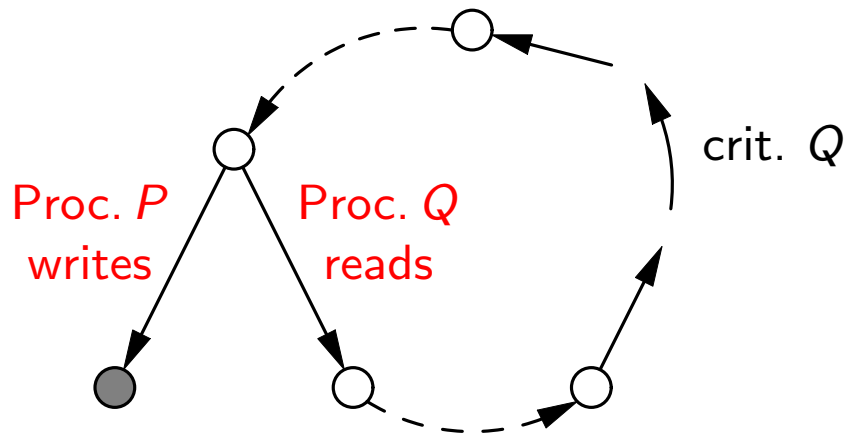


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