



UC SANTA CRUZ

Inductive diagrams for causal reasoning

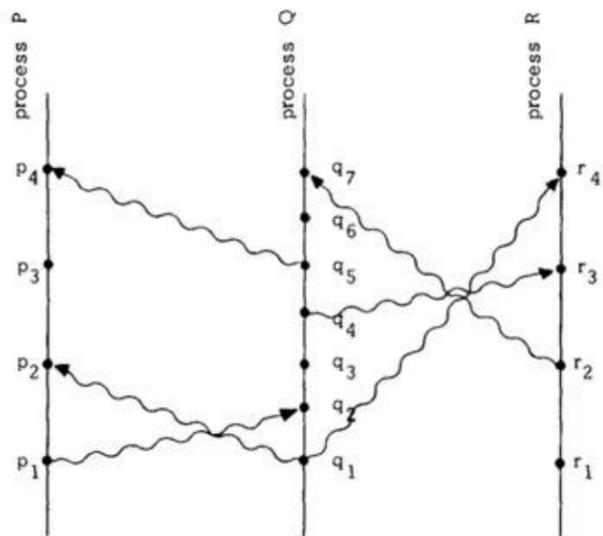
Jonathan Castello (UC Santa Cruz)

Patrick Redmond (UC Santa Cruz)

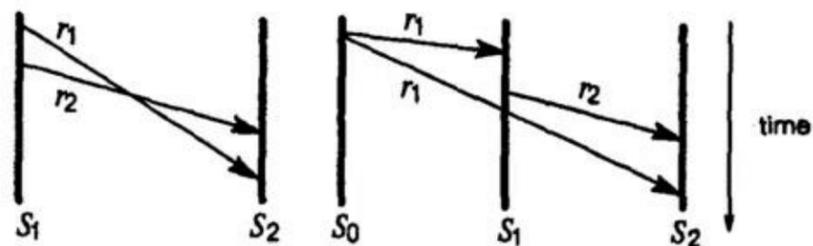
Lindsey Kuper (UC Santa Cruz)

OOPSLA'24

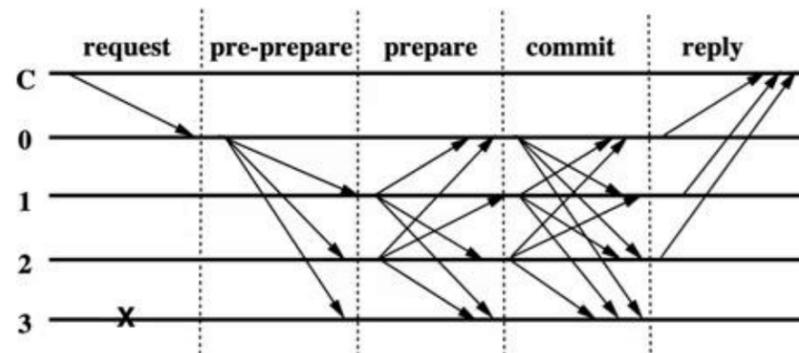
2024-10-23



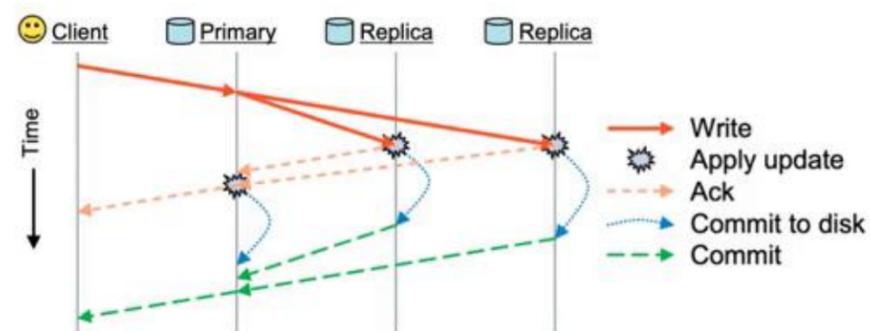
[Lamport 1978]



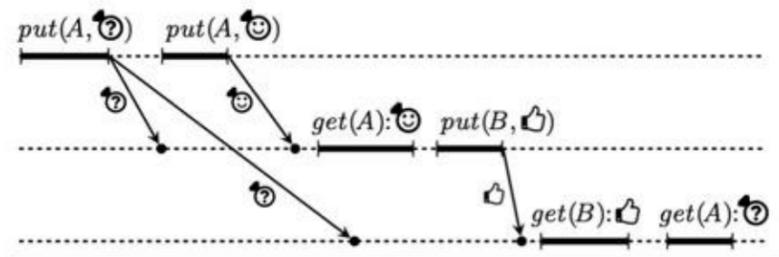
[Ellis and Gibbs 1989]



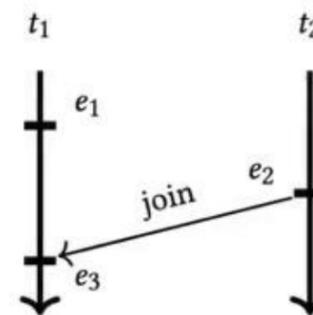
[Castro and Liskov 1999]



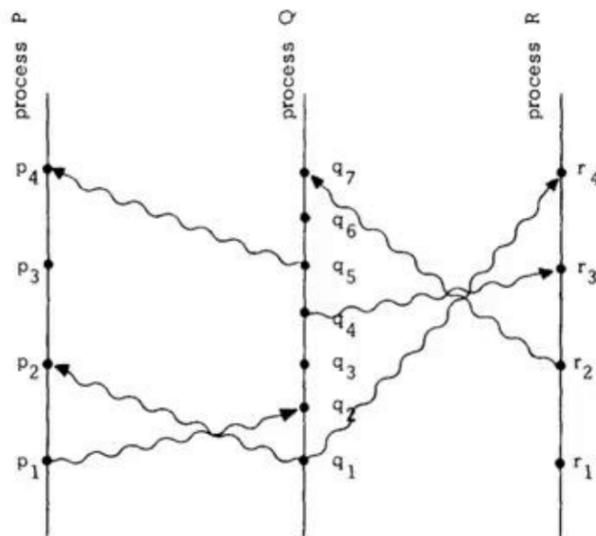
[Weil et al. 2006]



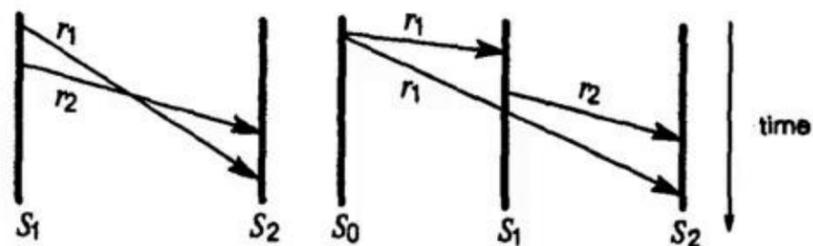
[Lesani et al. 2016]



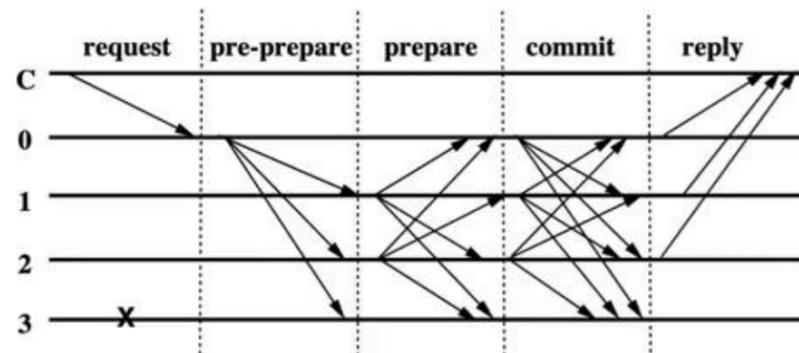
[Mathur et al. 2022]



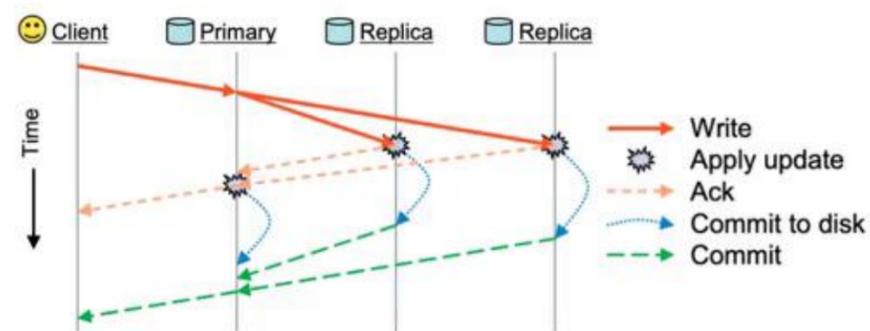
[Lamport 1978]



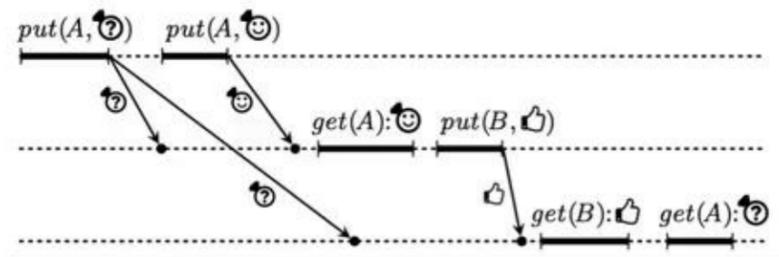
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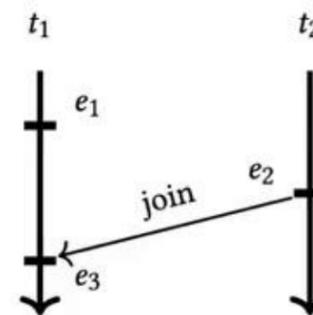
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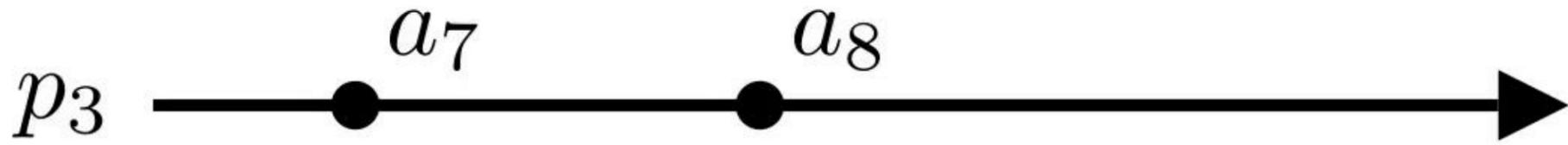
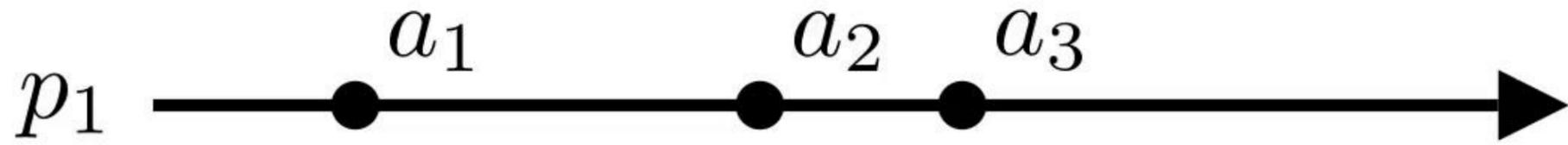
[Mathur et al. 2022]

p_1 →

p_2 →

p_3 →

$$p = \{p_1, p_2, p_3\}$$

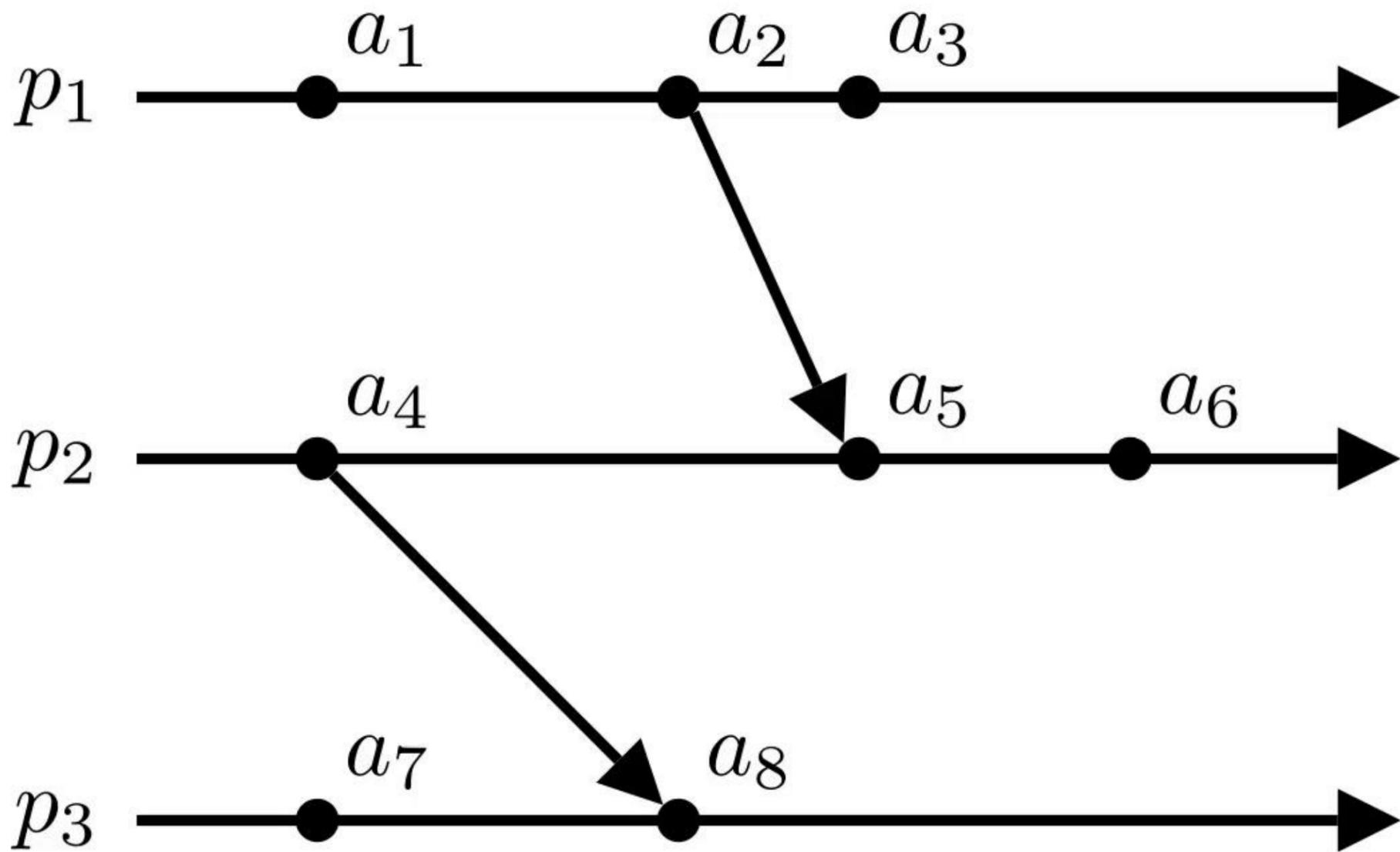


$$p = \{p_1, p_2, p_3\}$$

$$p_1 = \langle a_1, a_2, a_3 \rangle$$

$$p_2 = \langle a_4, a_5, a_6 \rangle$$

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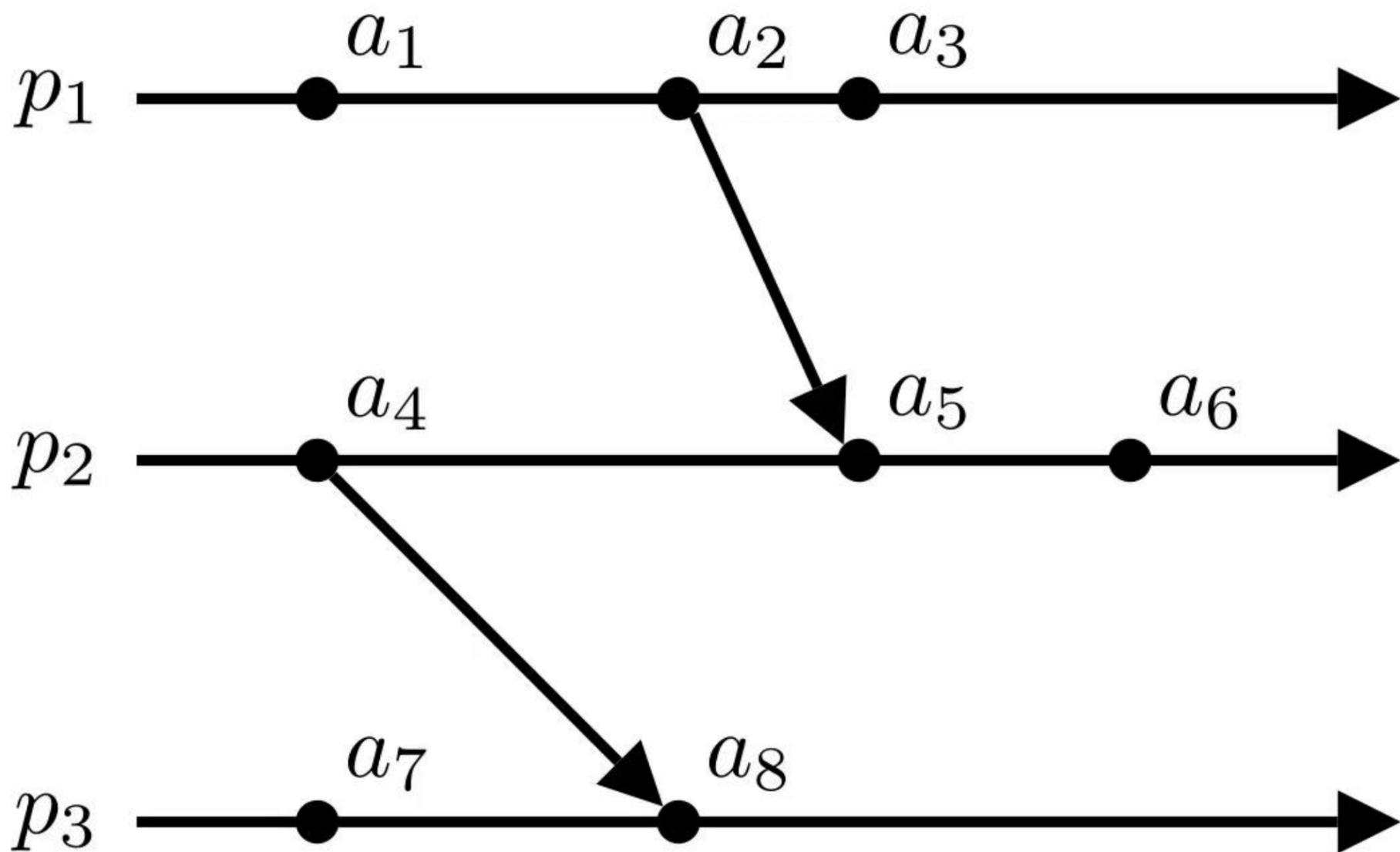
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$$m = \{(a_2, a_5), (a_4, a_8)\}$$



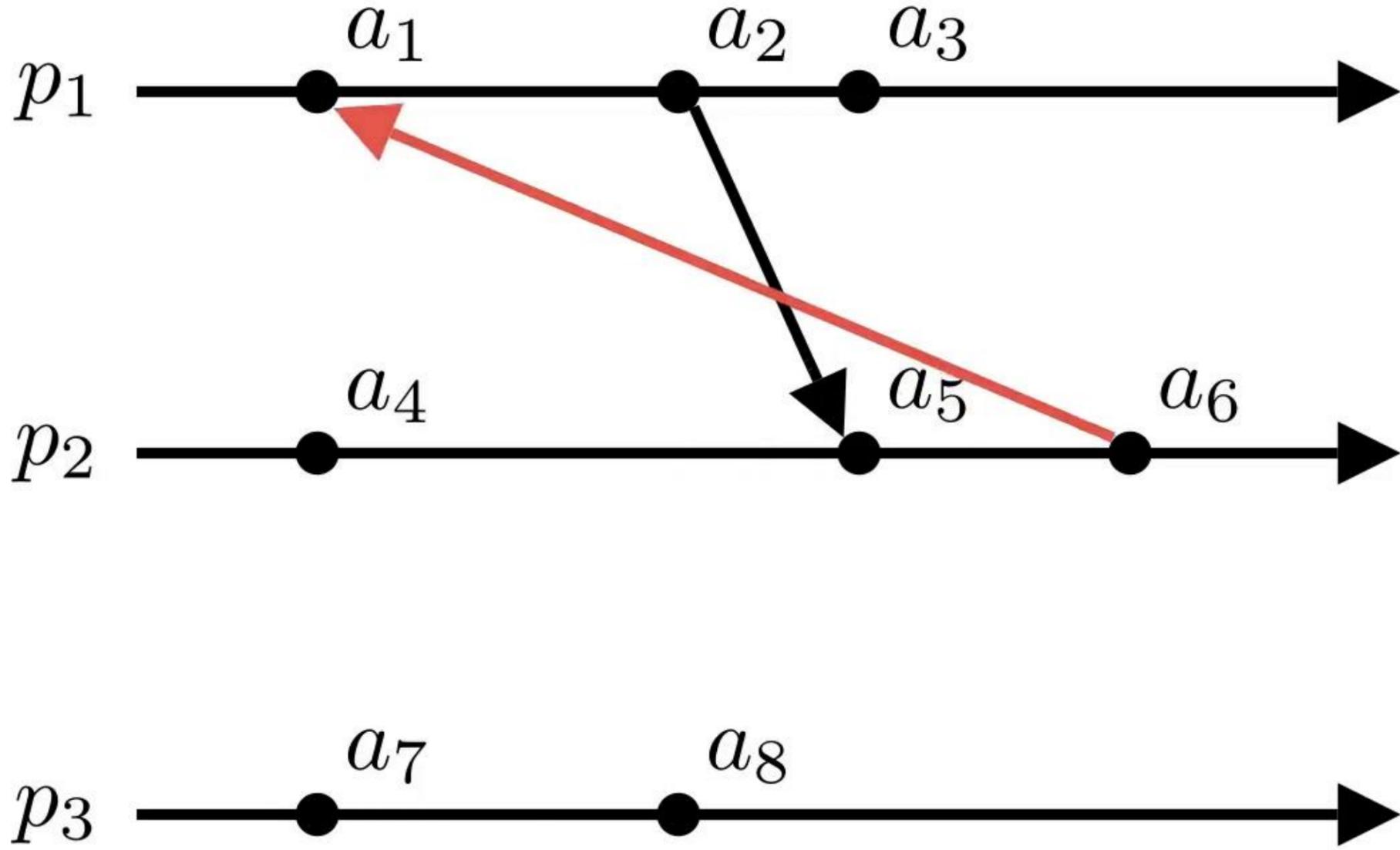
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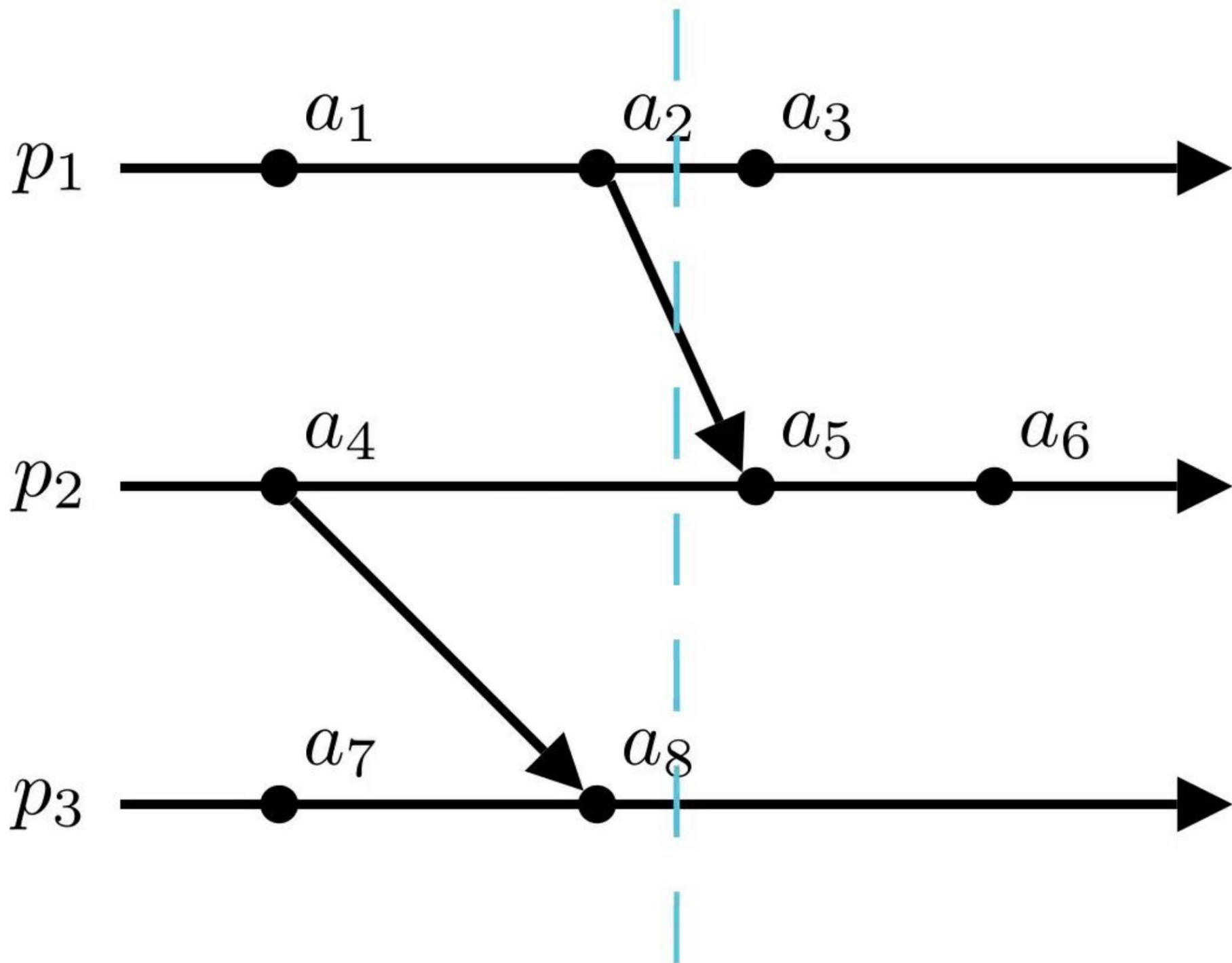
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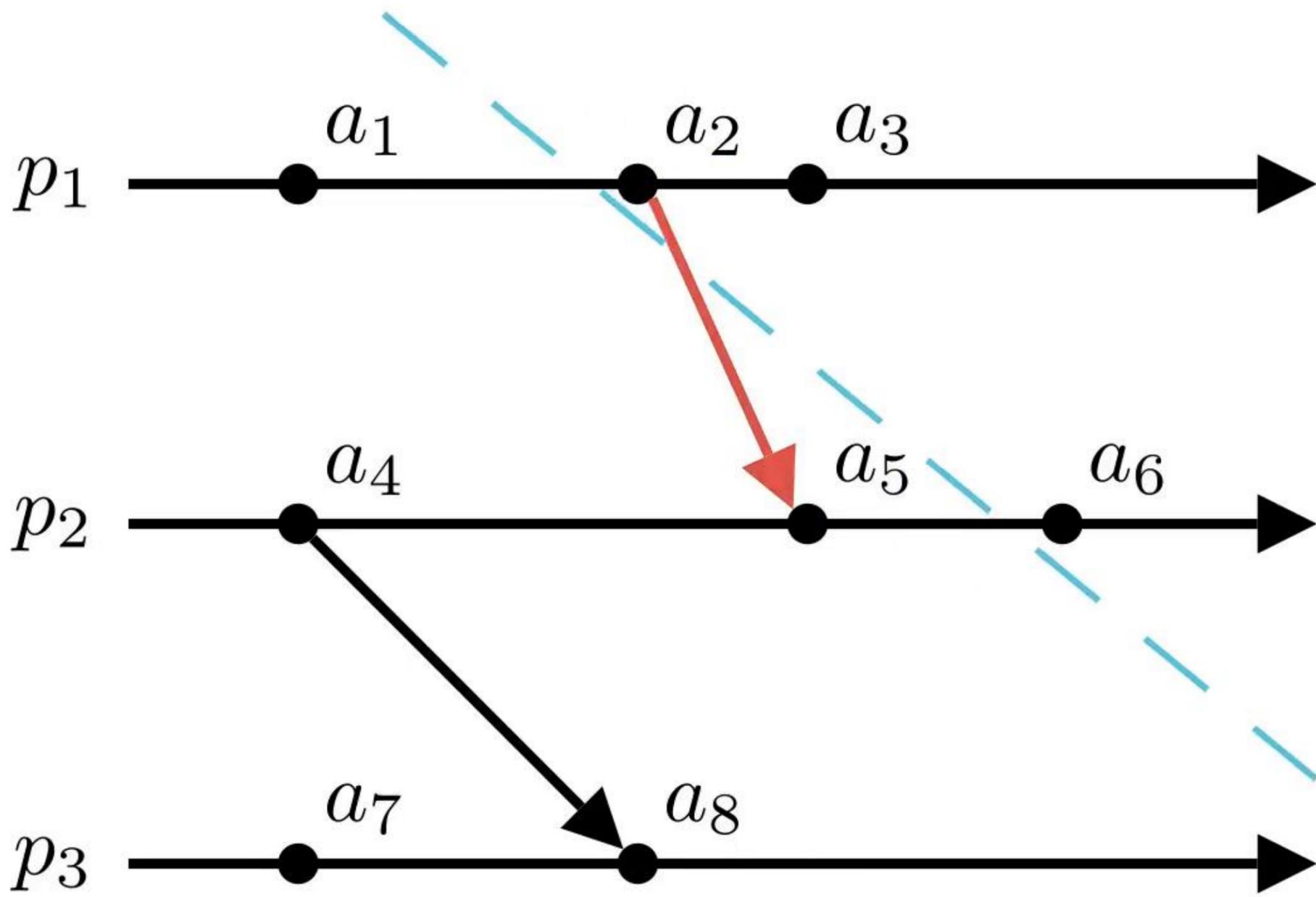
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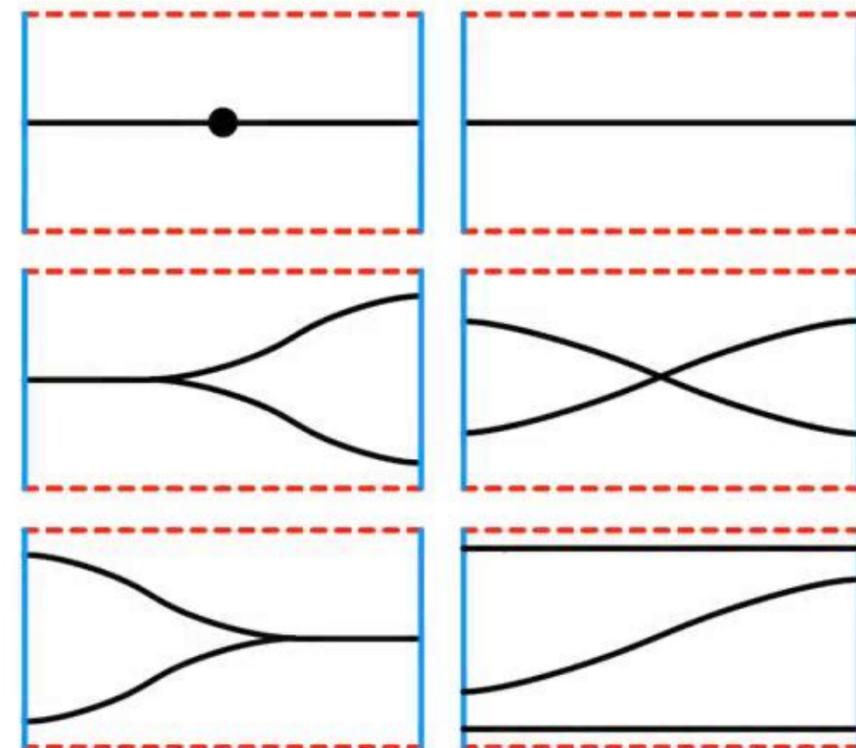
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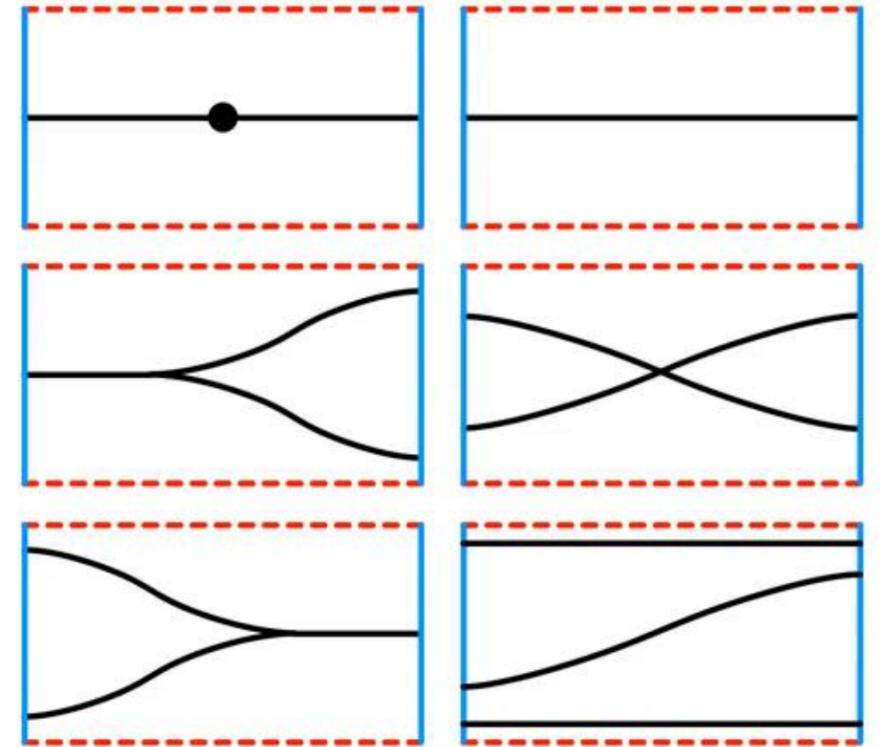
```
1 data BinaryTree : Type where
2   leaf : BinaryTree
3   node : (l r : BinaryTree) → BinaryTree
4
5 depth : BinaryTree → ℕ
6 depth leaf = 0
7 depth (node l r) = max (depth l) (depth r)
```

Causal Separation Diagrams



Causal Separation Diagrams

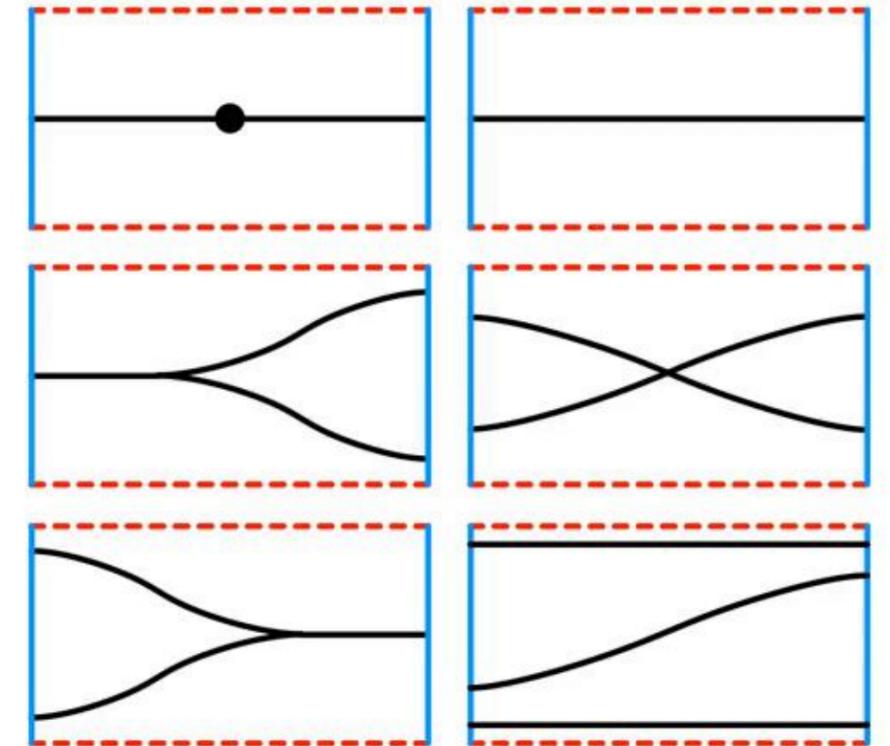
Causal by construction



Causal Separation Diagrams

Causal by construction

Inductively defined

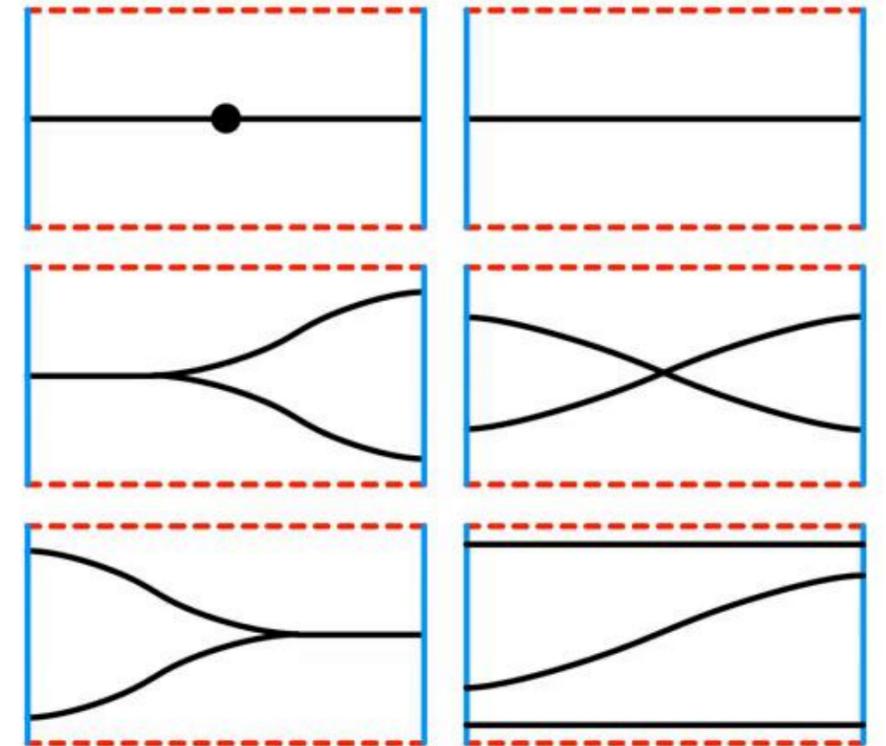


Causal Separation Diagrams

Causal by construction

Inductively defined

Graphical syntax



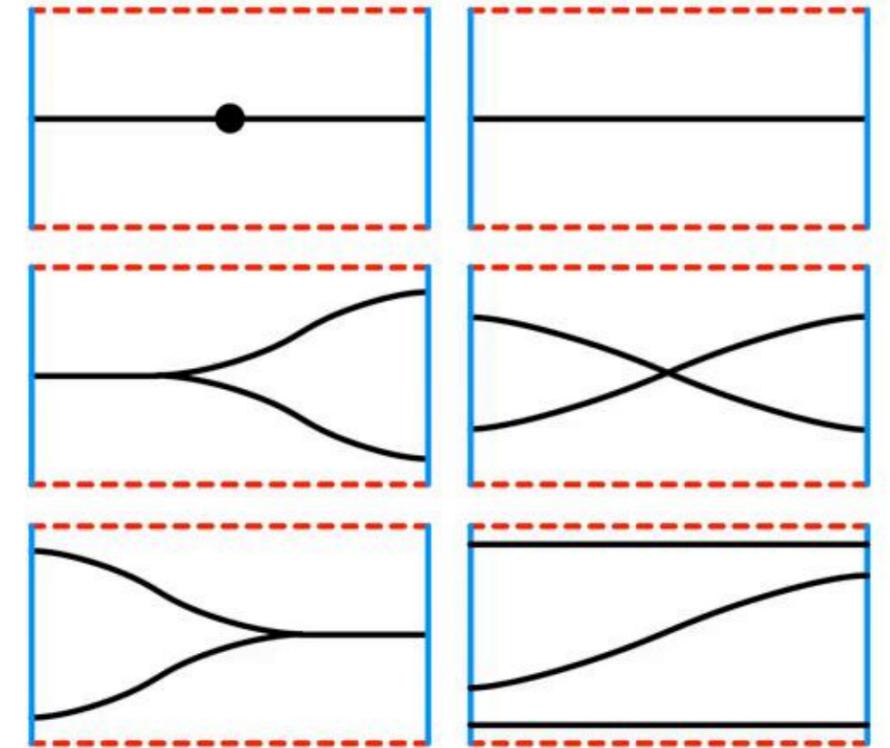
Causal Separation Diagrams

Causal by construction

Inductively defined

Graphical syntax

Compositional semantics



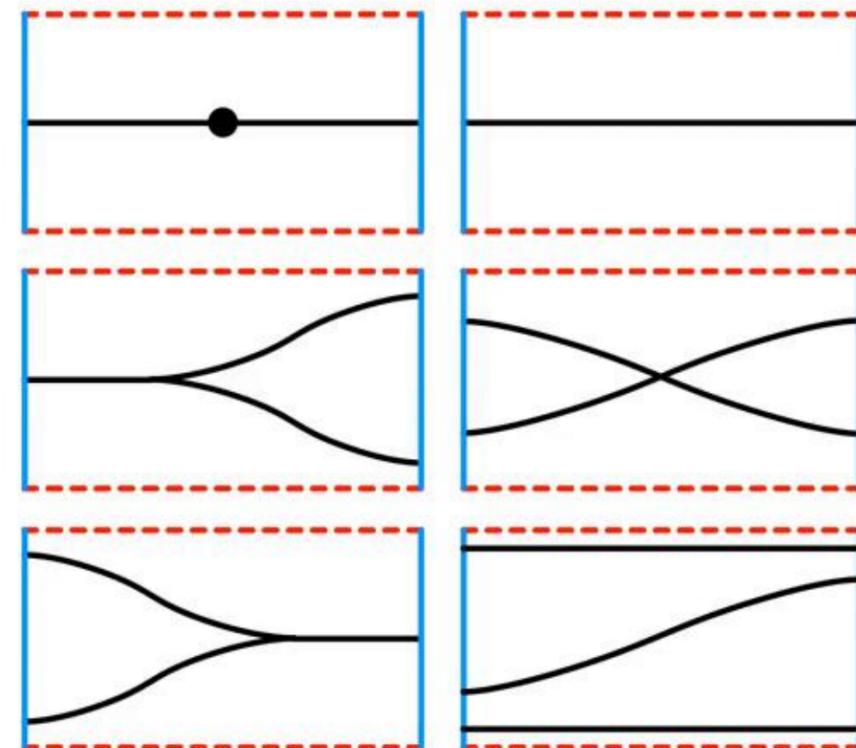
Causal Separation Diagrams

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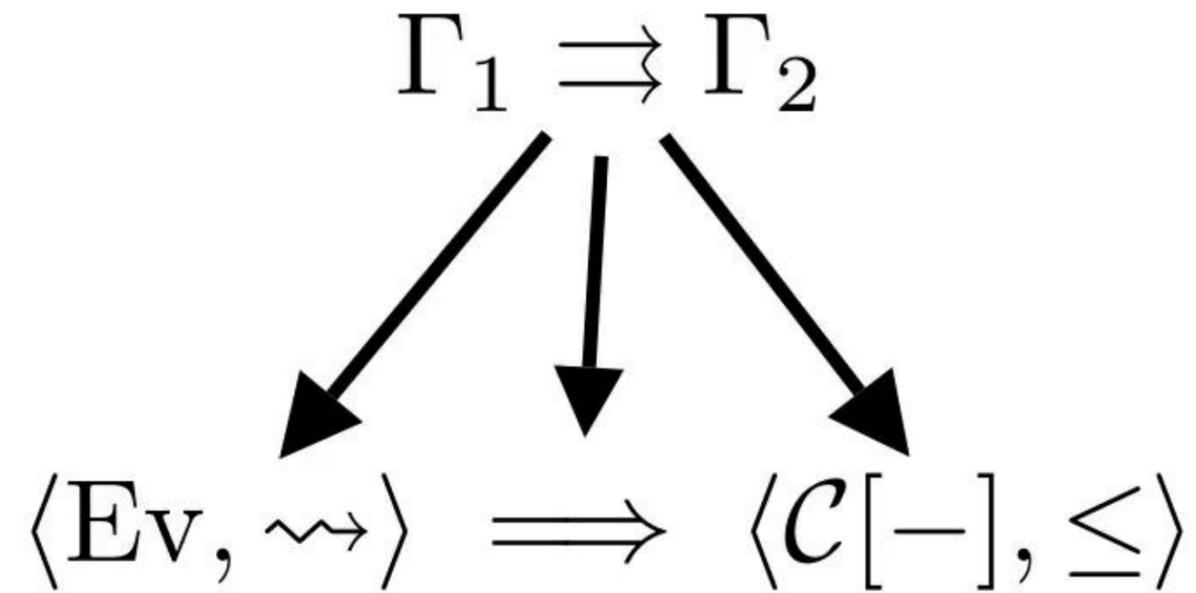
Compositional semantics



What's in the paper?

$$\Gamma_1 \Rightarrow \Gamma_2$$

What's in the paper?



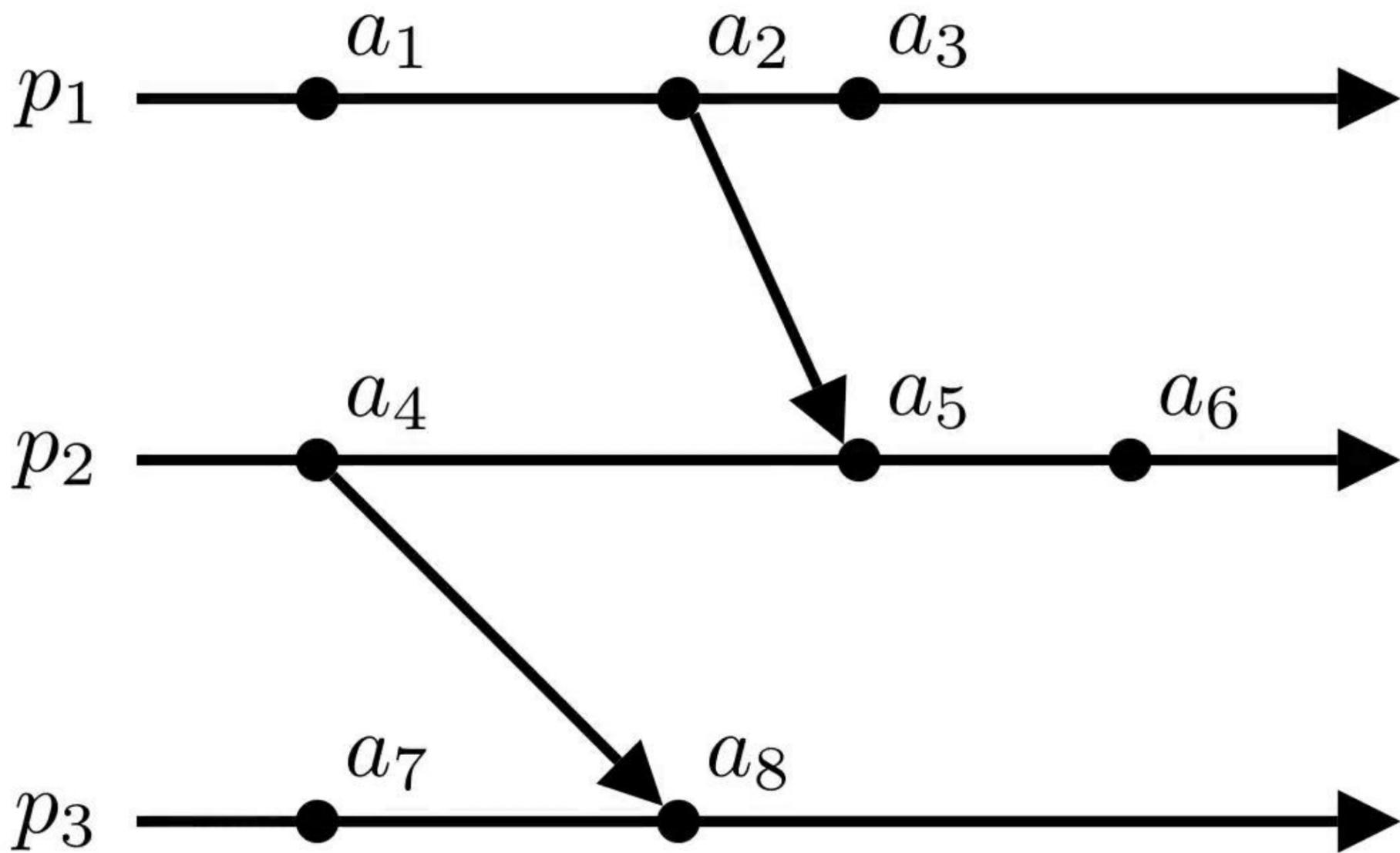
<https://github.com/lsc-ucsc/csds>

Causal by construction

Inductively defined

Graphical syntax

Compositional semantics



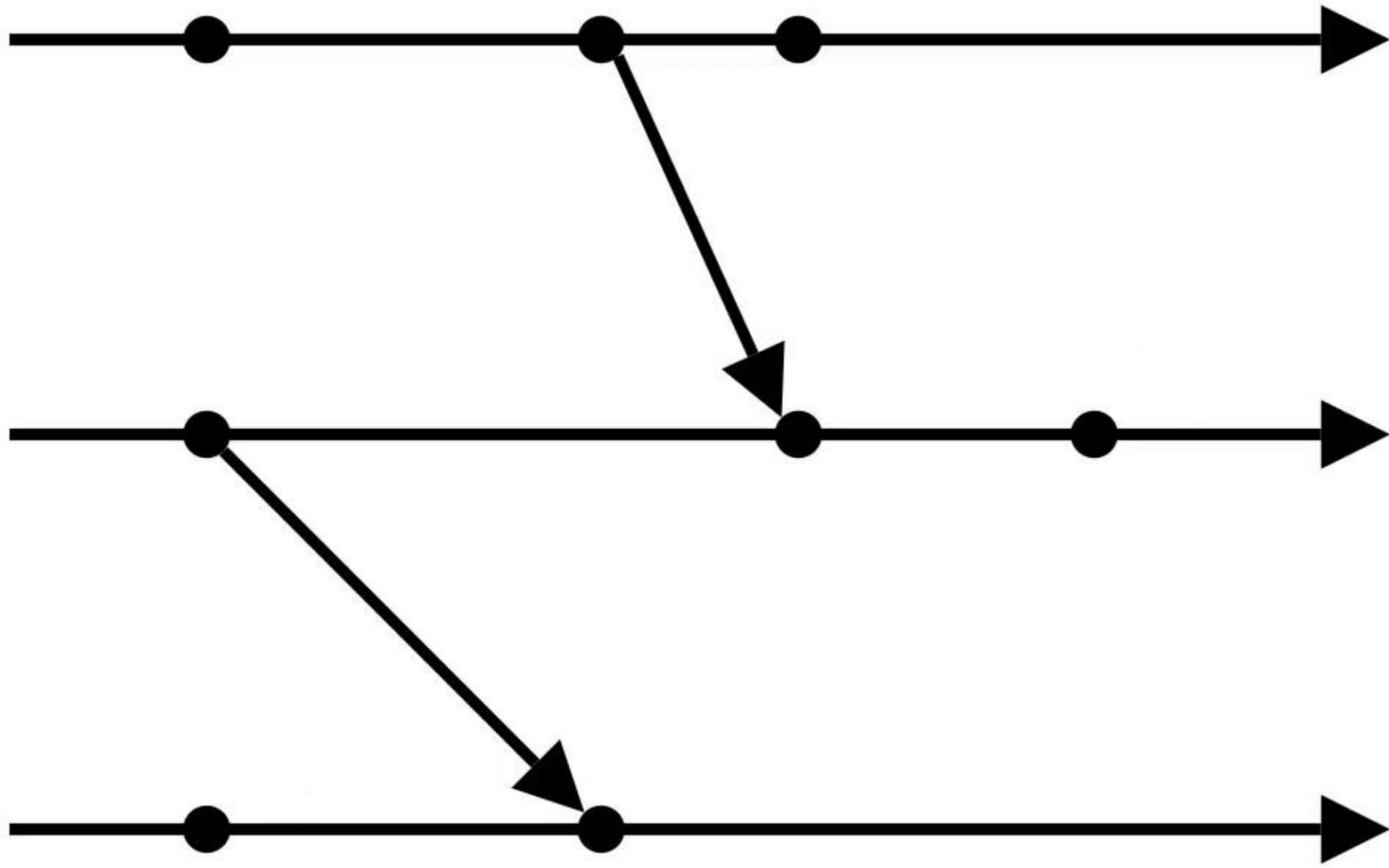
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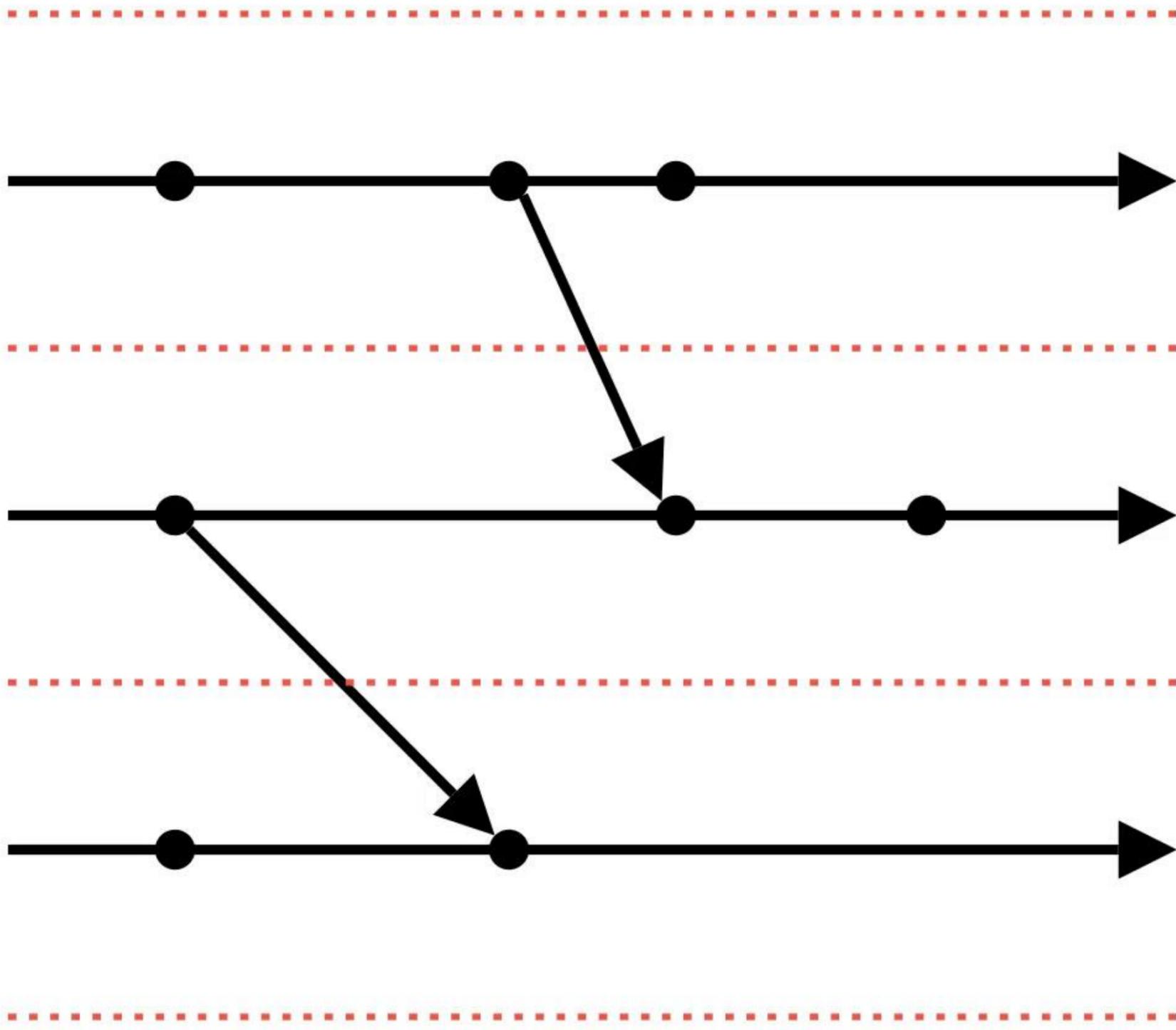
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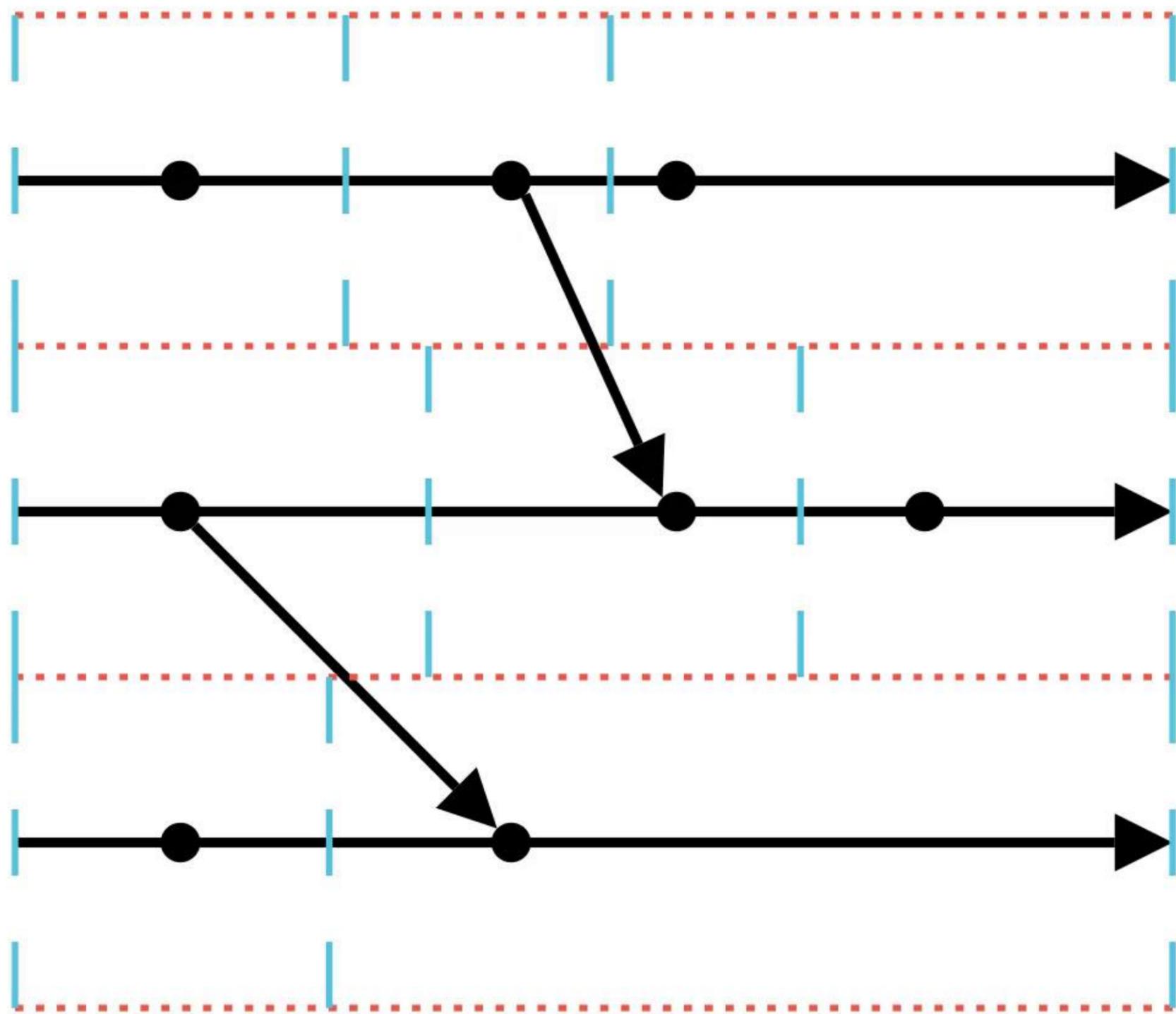
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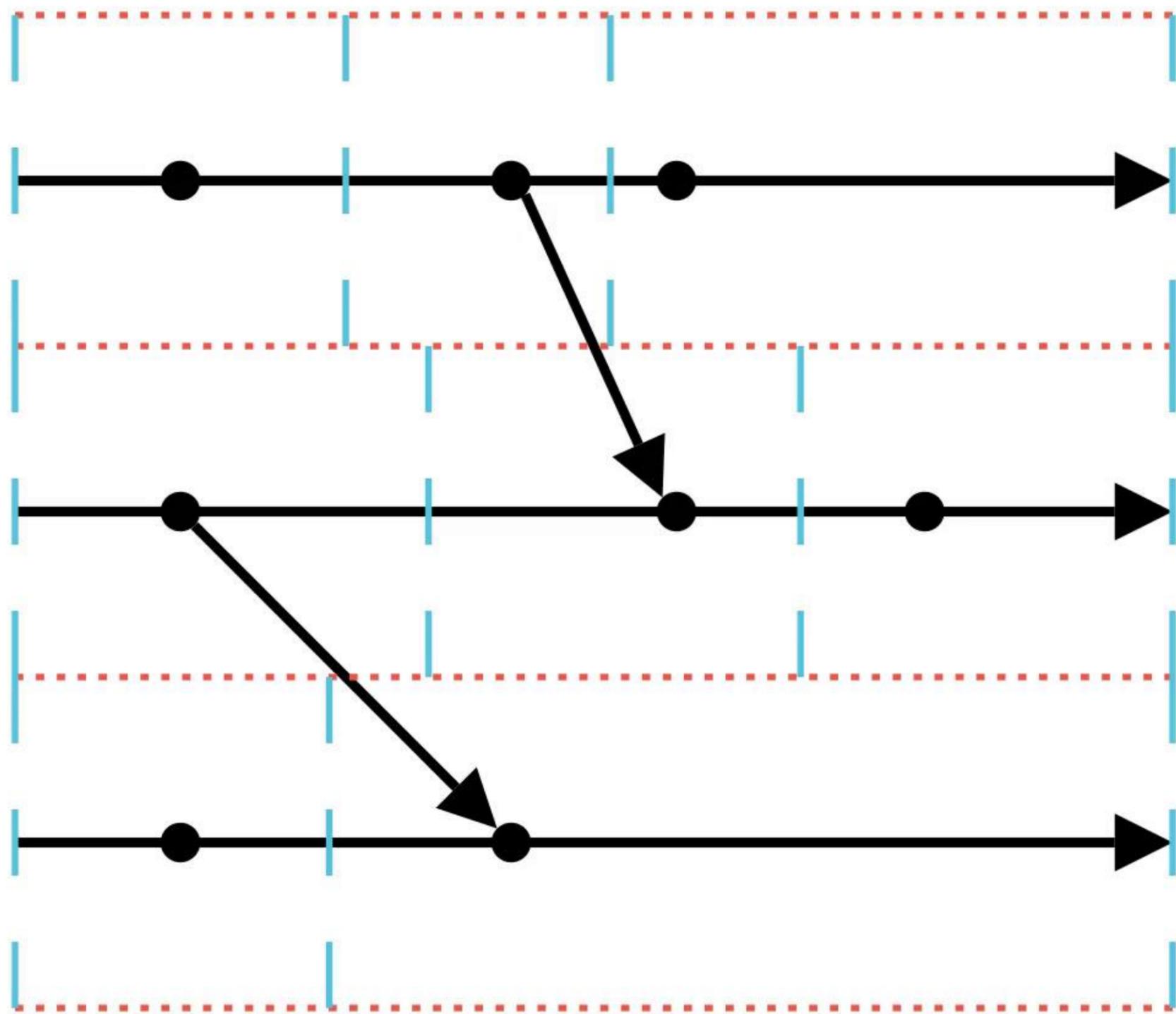
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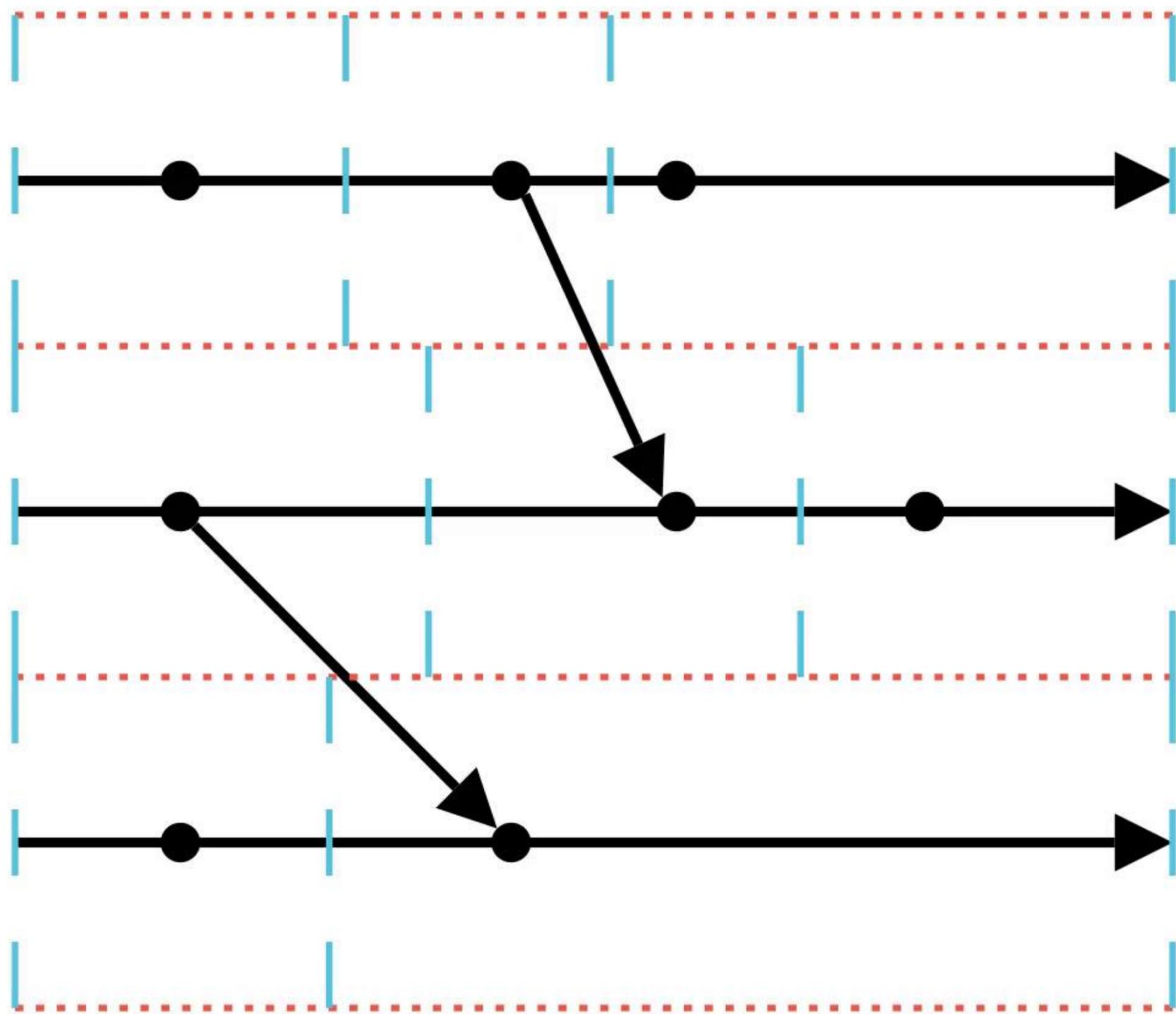
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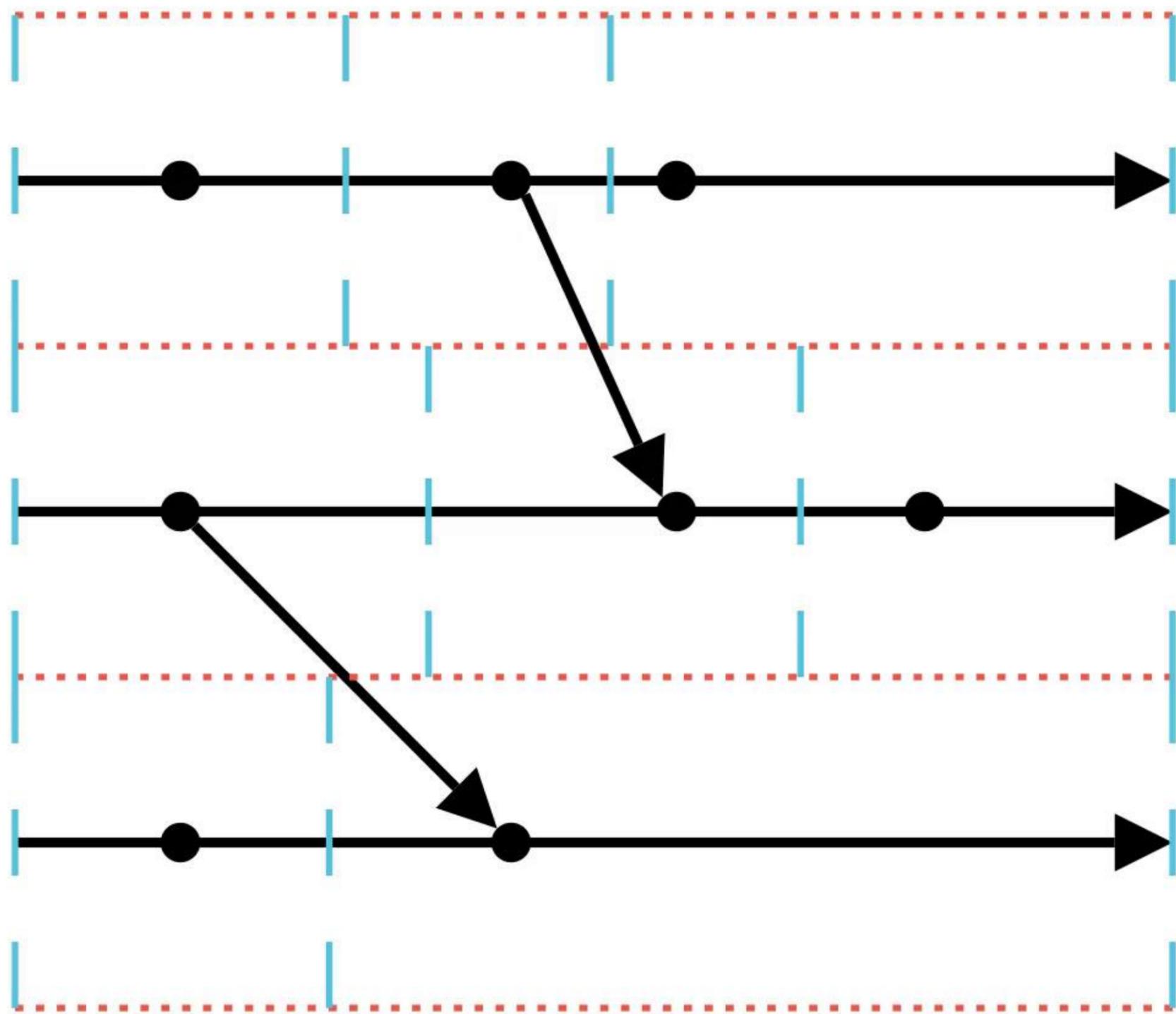
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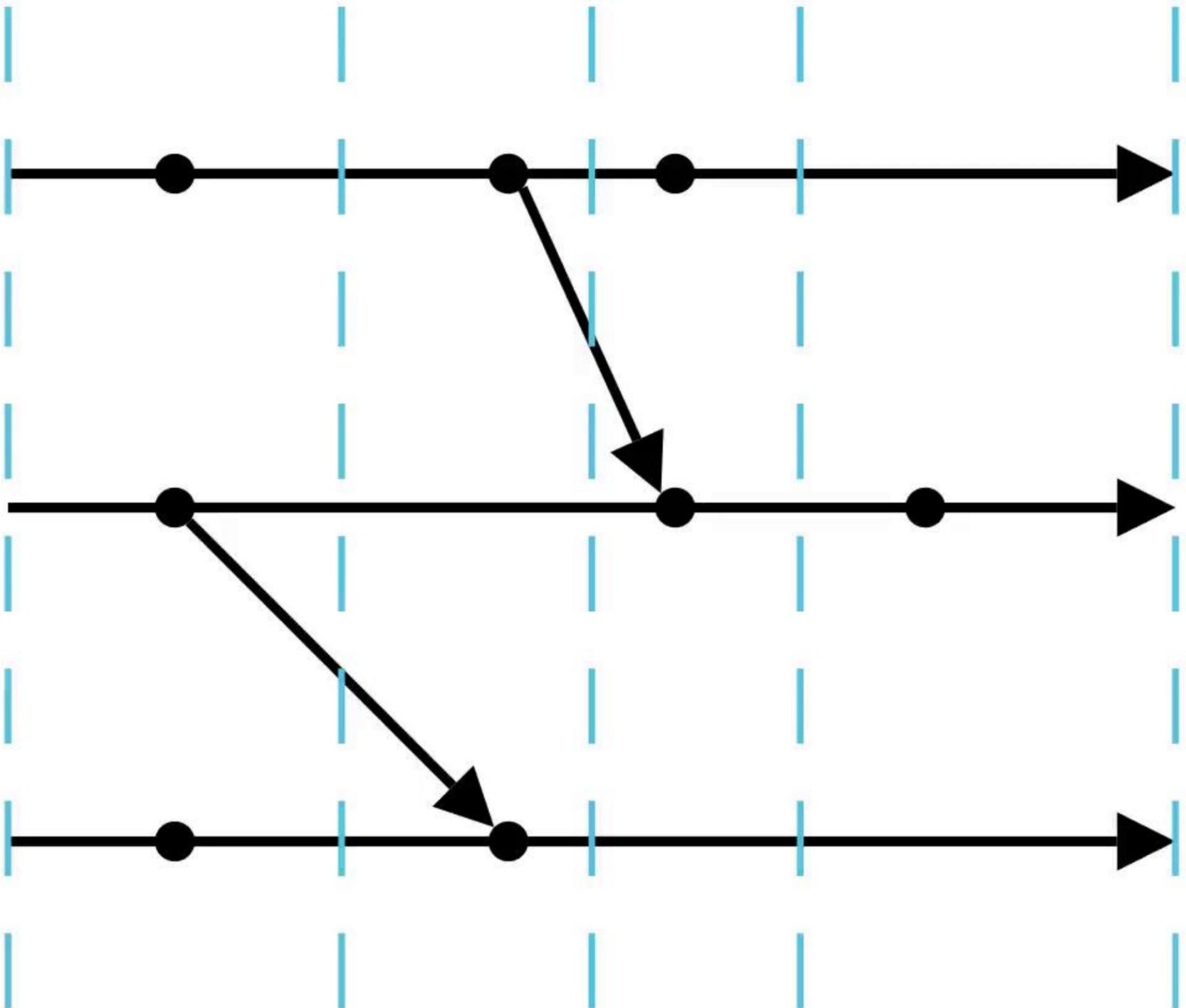
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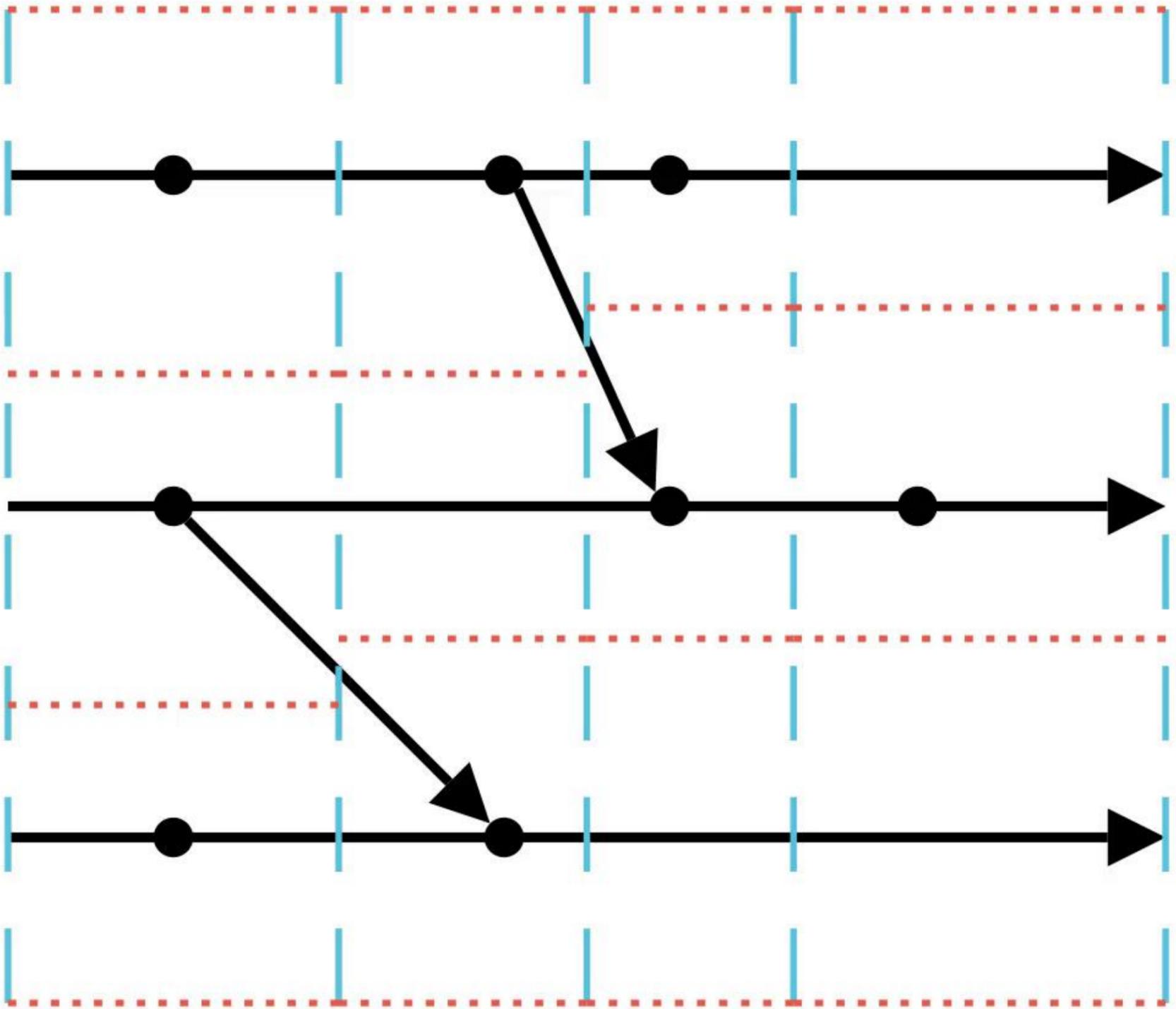
$$p_1 = \langle a_1, a_2, a_3 \rangle$$

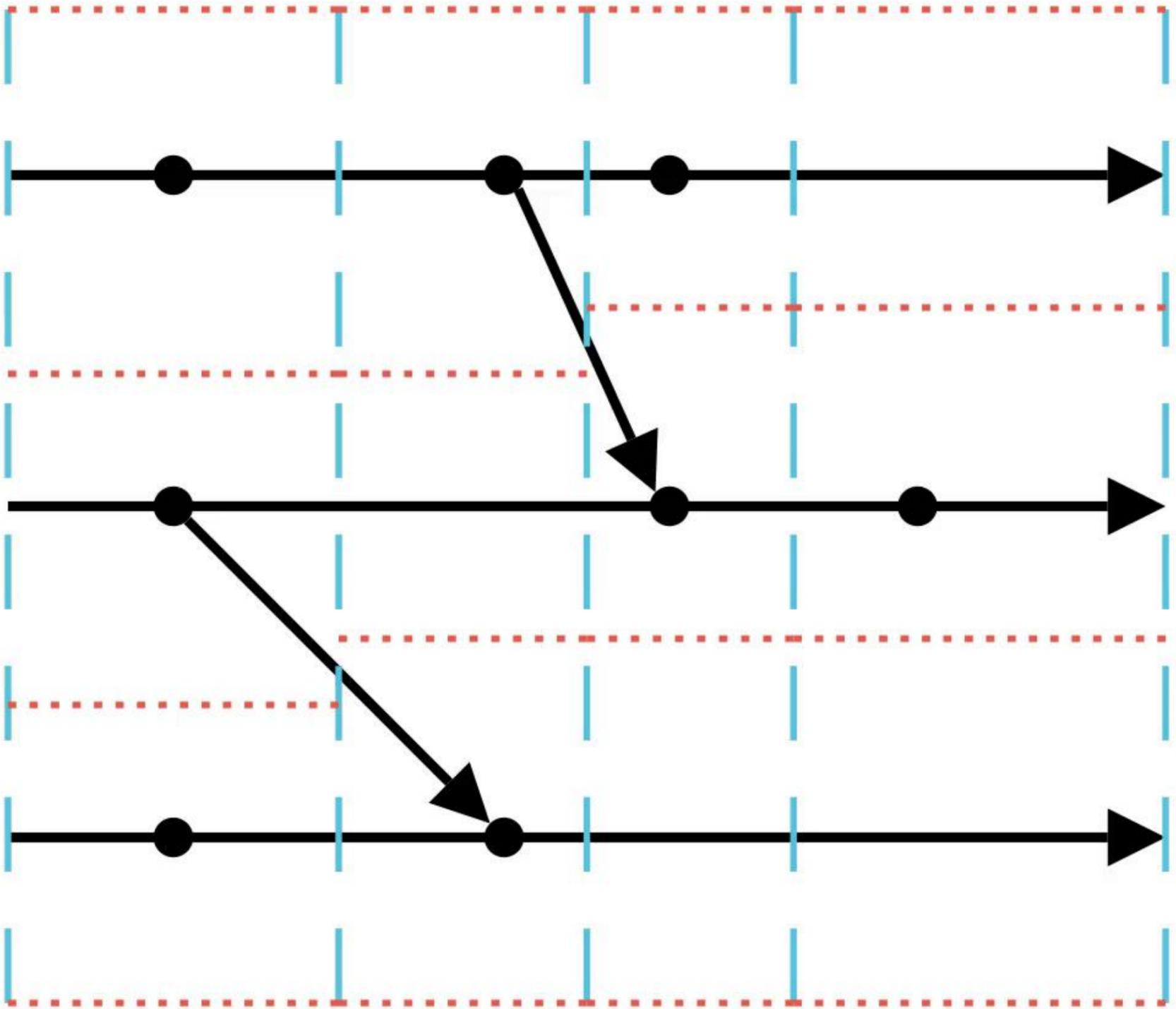
$$p_2 = \langle a_4, a_5, a_6 \rangle$$

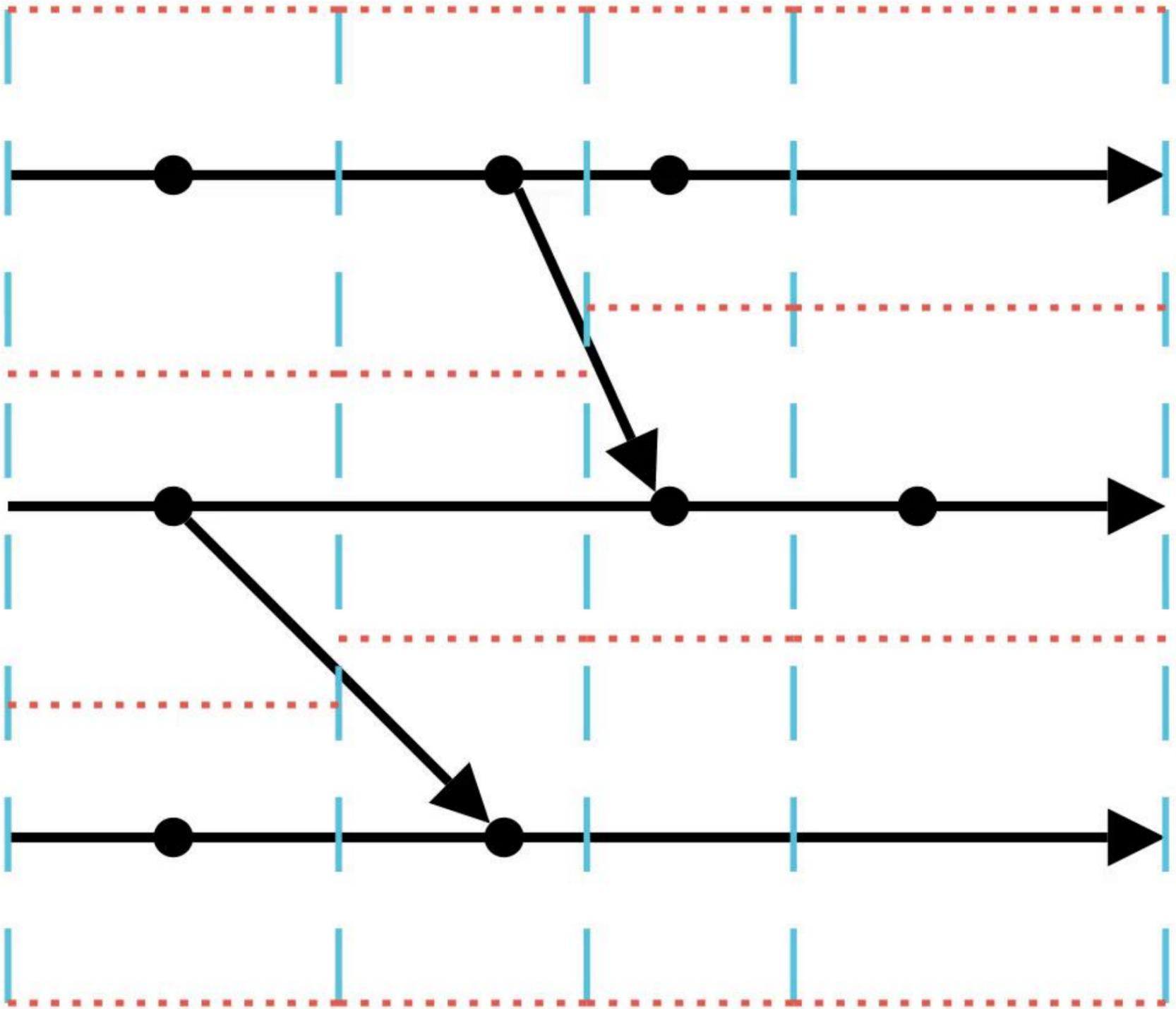
$$p_3 = \langle a_7, a_8 \rangle$$

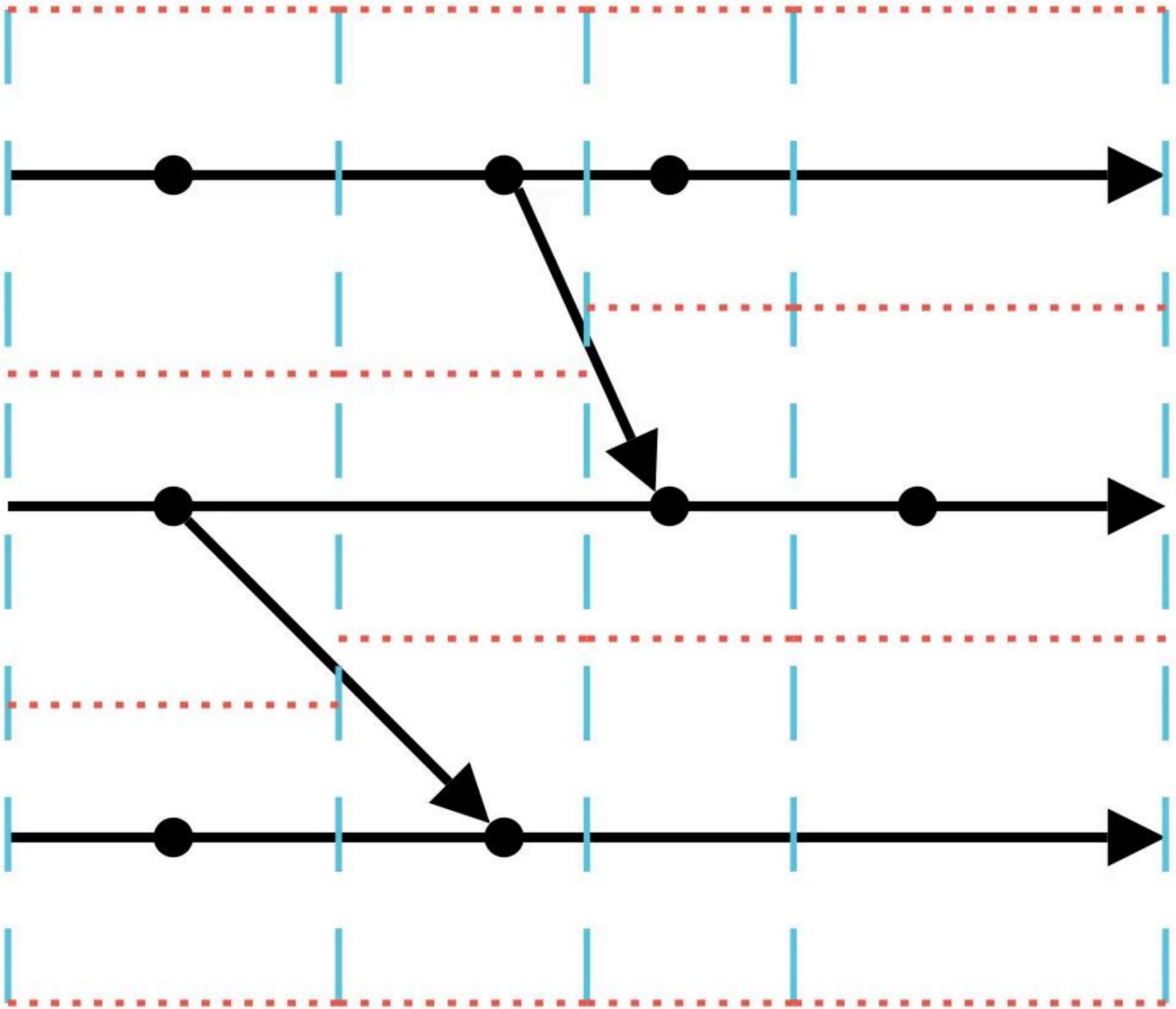
$$m = \{(a_2, a_5), (a_4, a_8)\}$$

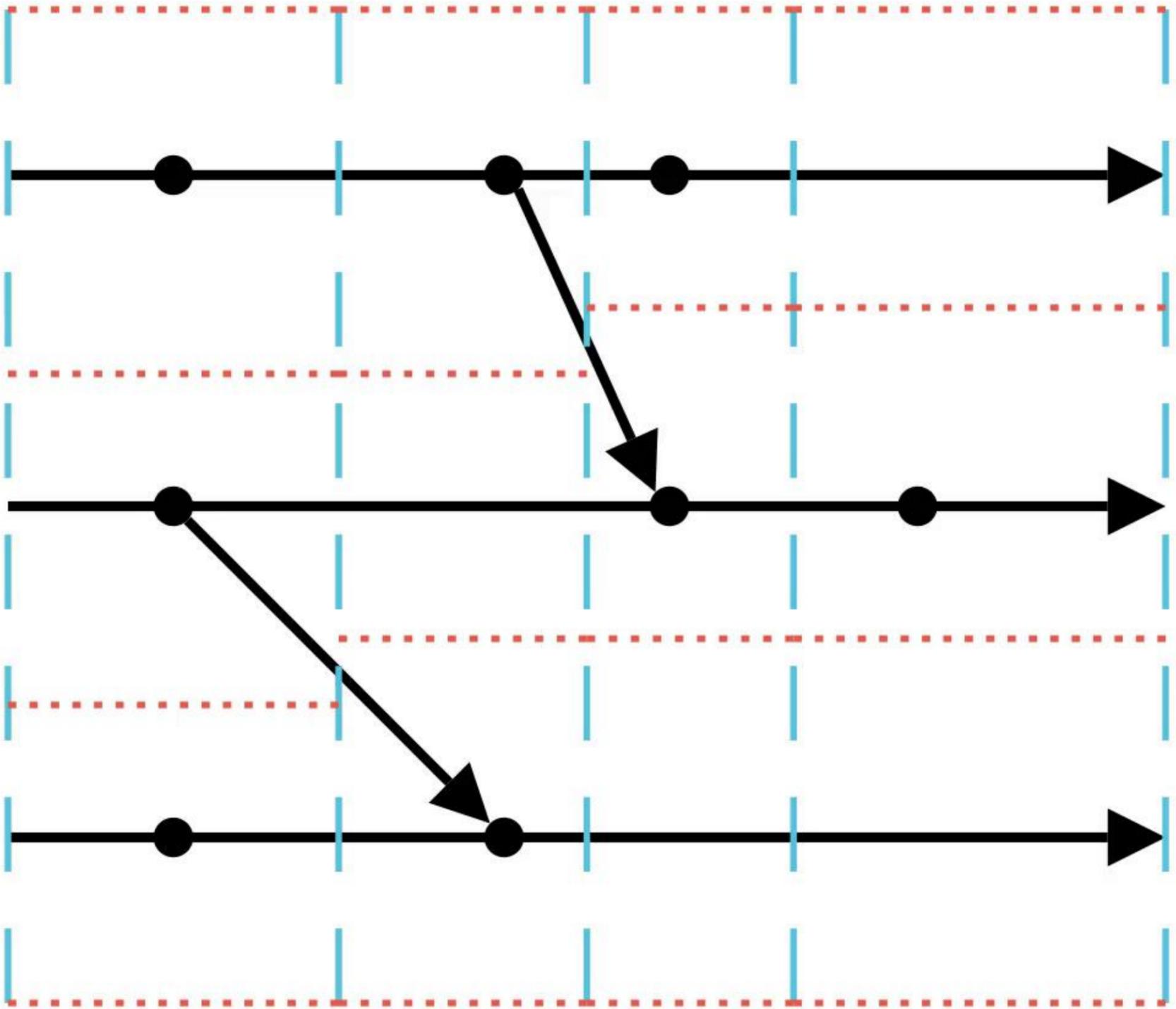












Causal by construction

Inductively defined

Graphical syntax

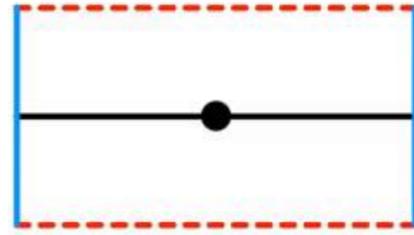
Compositional semantics

Causal by construction

Inductively defined

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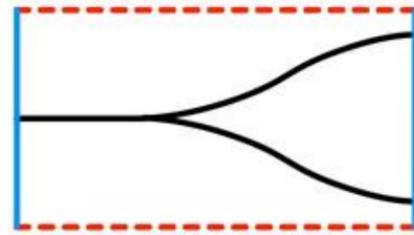
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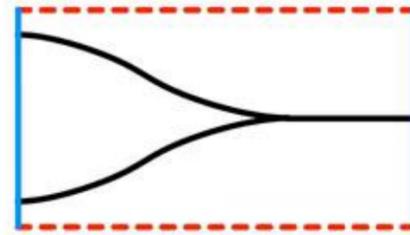
tick



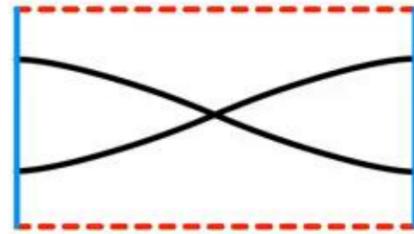
noop



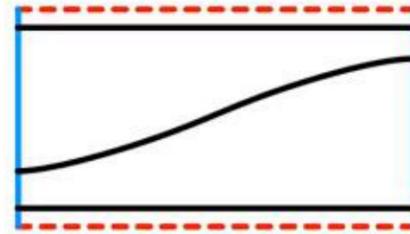
fork



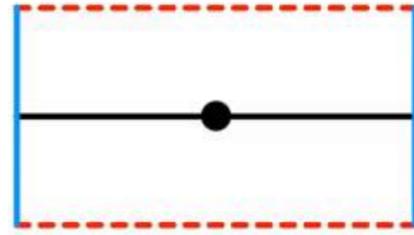
join



swap



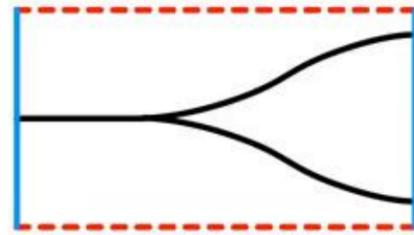
assoc



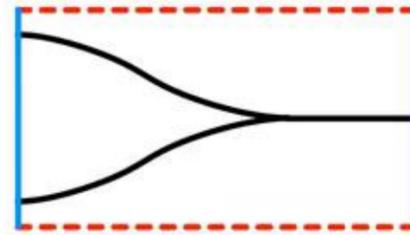
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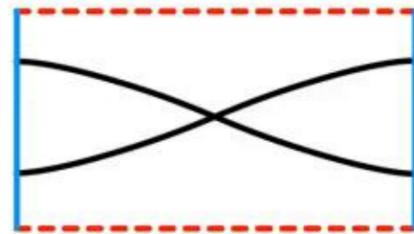
noop



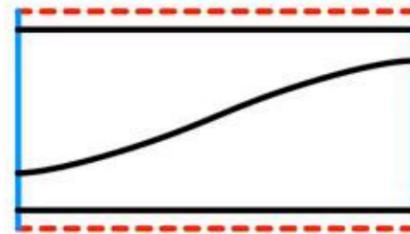
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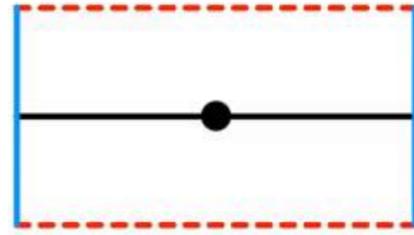
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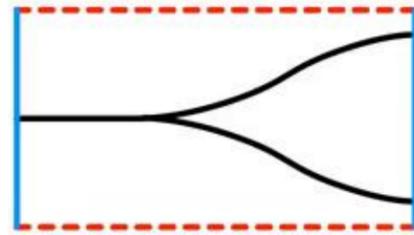
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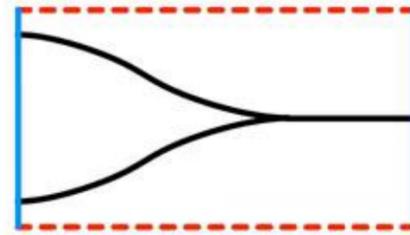
tick



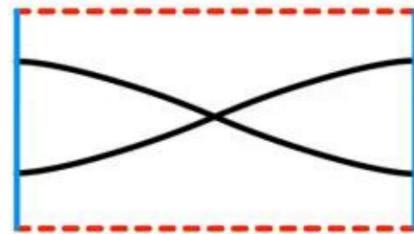
noop



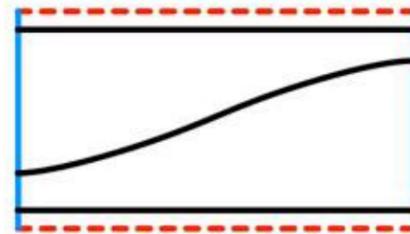
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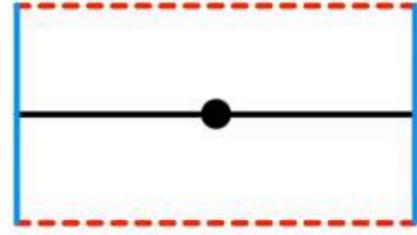
join



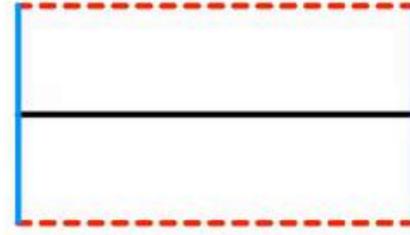
swap



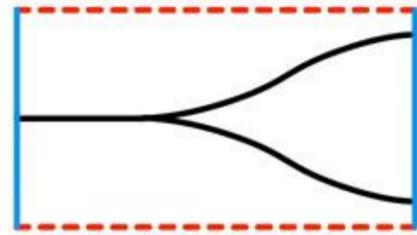
assoc



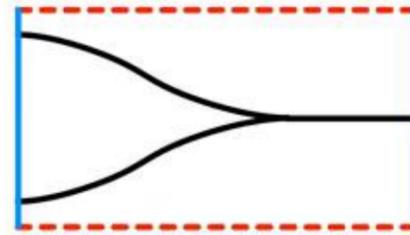
tick : $[\tau_1] \multimap [\tau_2]$



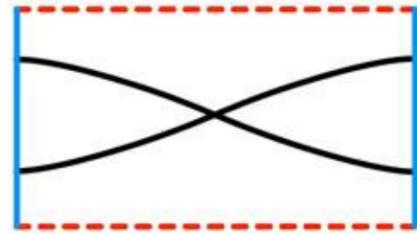
noop : $\Gamma \multimap \Gamma$



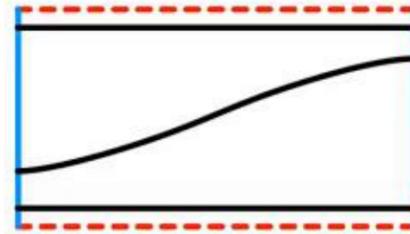
fork : $[\tau \times \tau']$
 $\multimap [\tau] \otimes [\tau']$



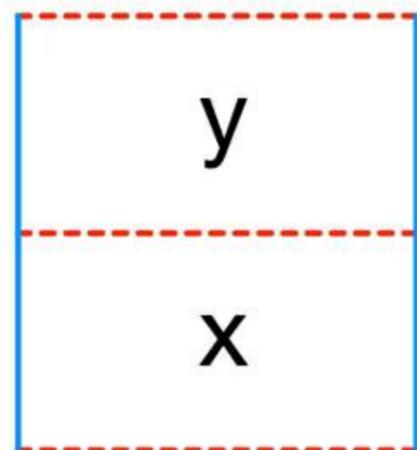
join : $[\tau] \otimes [\tau']$
 $\multimap [\tau \times \tau']$



swap : $[\tau] \otimes [\tau']$
 $\multimap [\tau'] \otimes [\tau]$



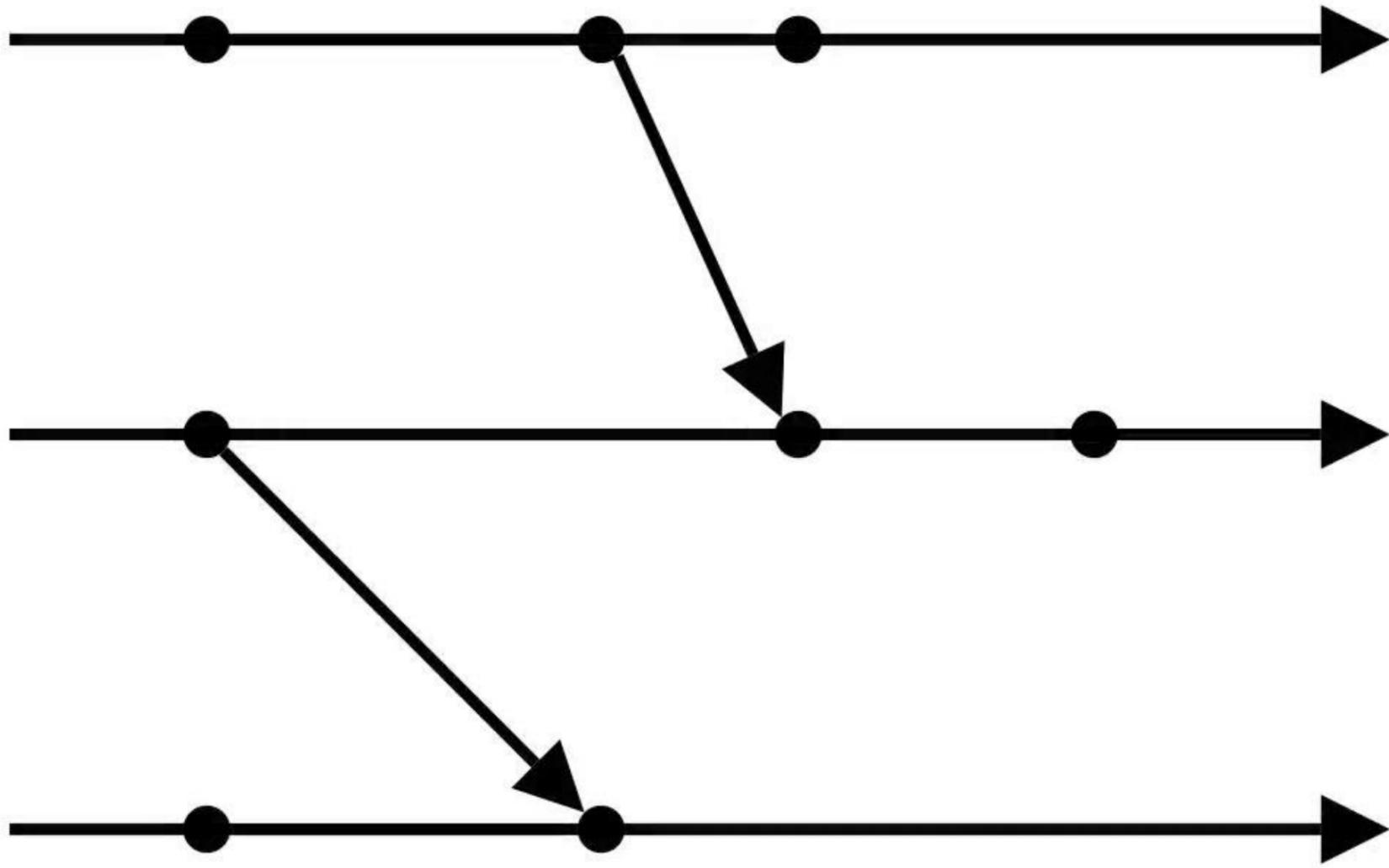
assoc : $([\tau] \otimes [\tau']) \otimes [\tau'']$
 $\multimap [\tau] \otimes ([\tau'] \otimes [\tau''])$

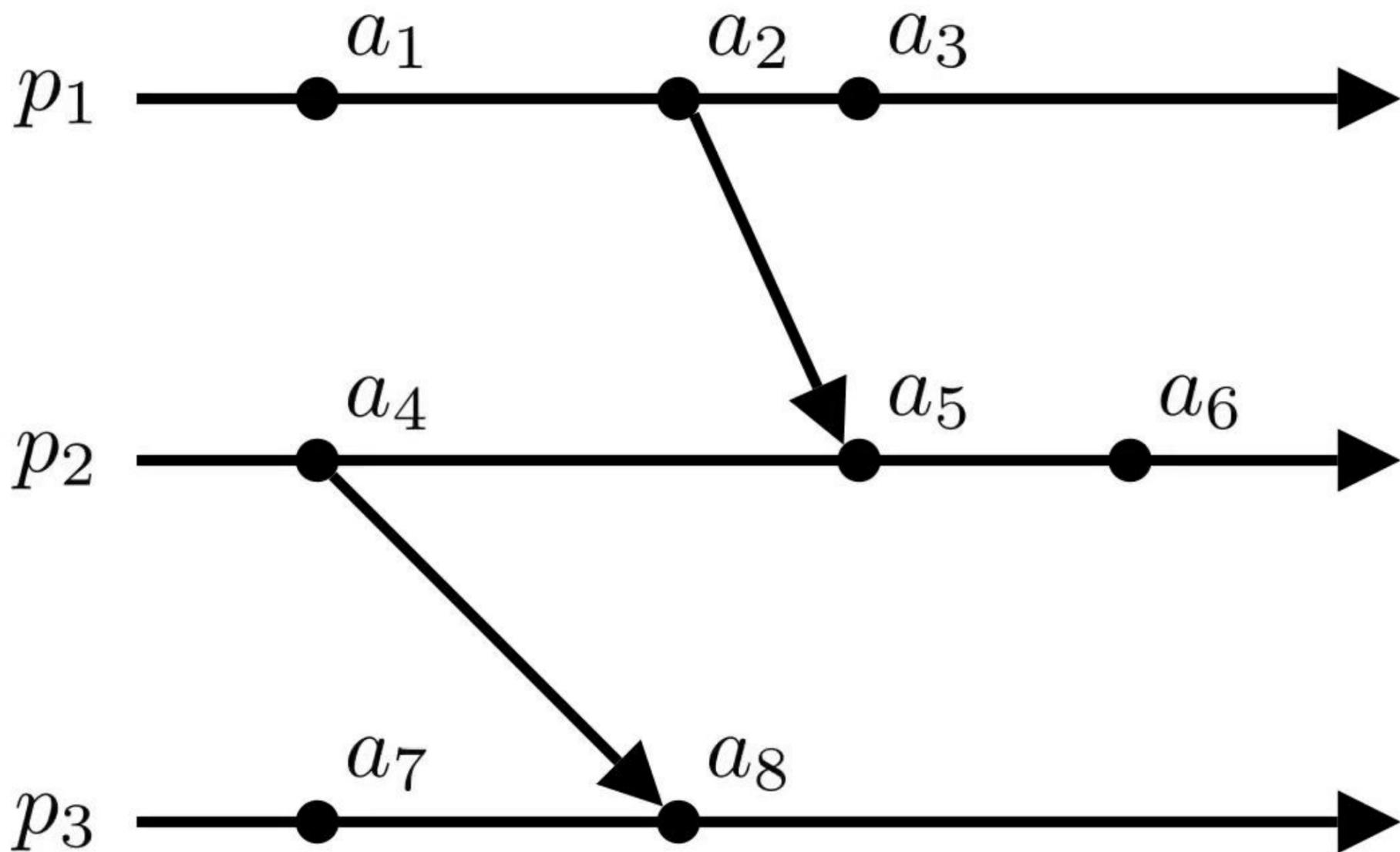


$$\begin{aligned}
 (x \parallel y) &: (\Gamma_1 \multimap \Gamma_2) \\
 &\rightarrow (\Gamma'_1 \multimap \Gamma'_2) \\
 &\rightarrow (\Gamma_1 \otimes \Gamma'_1 \multimap \Gamma_2 \otimes \Gamma'_2)
 \end{aligned}$$



$$\begin{aligned}
 (x ; y) &: (\Gamma_1 \rightrightarrows \Gamma_2) \\
 &\rightarrow (\Gamma_2 \multimap \Gamma_3) \\
 &\rightarrow (\Gamma_1 \rightrightarrows \Gamma_3)
 \end{aligned}$$





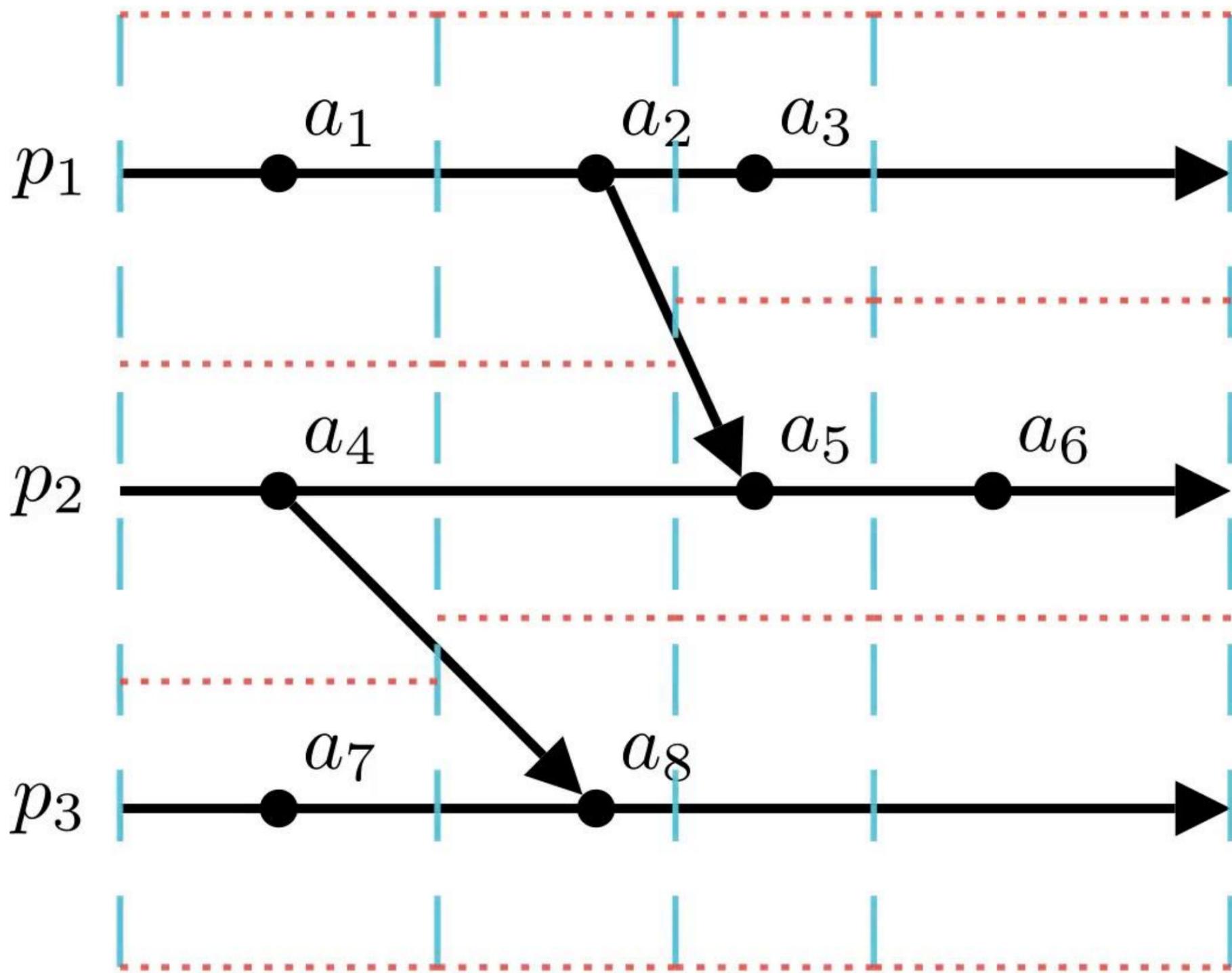
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$$p_3 = \langle a_7, a_8 \rangle$$

$$m = \{(a_2, a_5), (a_4, a_8)\}$$



$$p = \{p_1, p_2, p_3\}$$

$$p_1 = \langle a_1, a_2, a_3 \rangle$$

$$p_2 = \langle a_4, a_5, a_6 \rangle$$

$$p_3 = \langle a_7, a_8 \rangle$$

$$m = \{(a_2, a_5), (a_4, a_8)\}$$

tick || fork || tick;

join || noop || fork;

noop || join || tick;

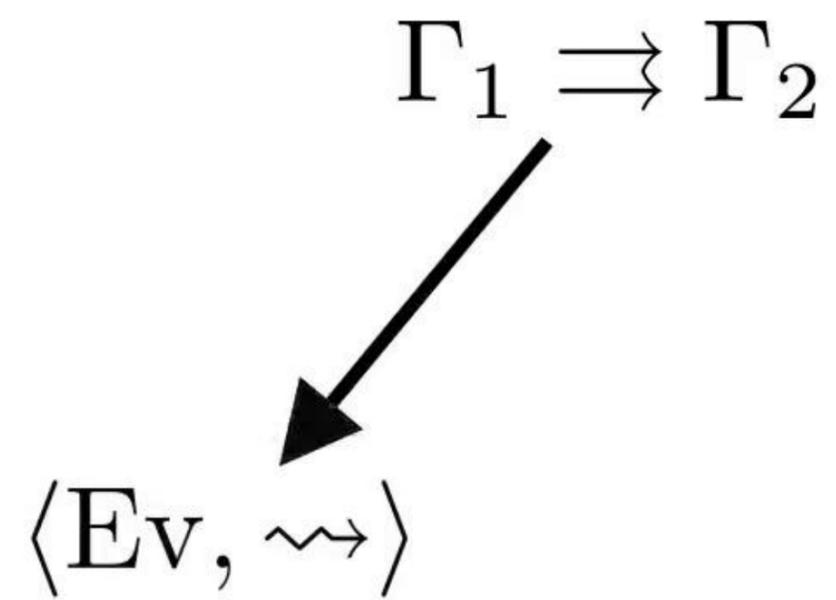
noop || tick || noop

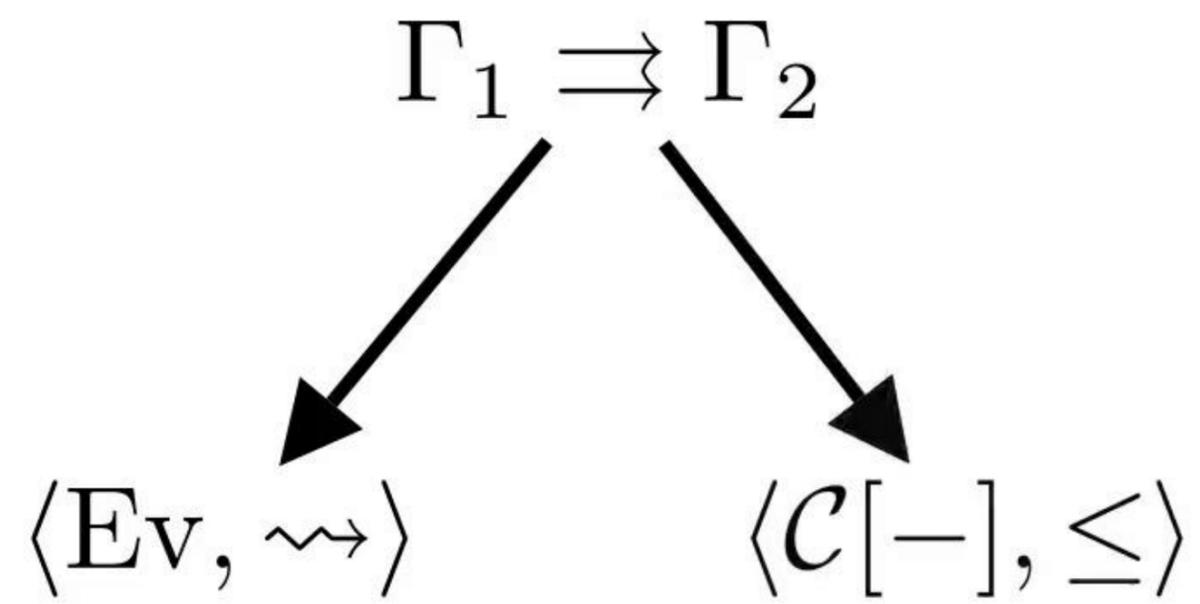
Causal by construction

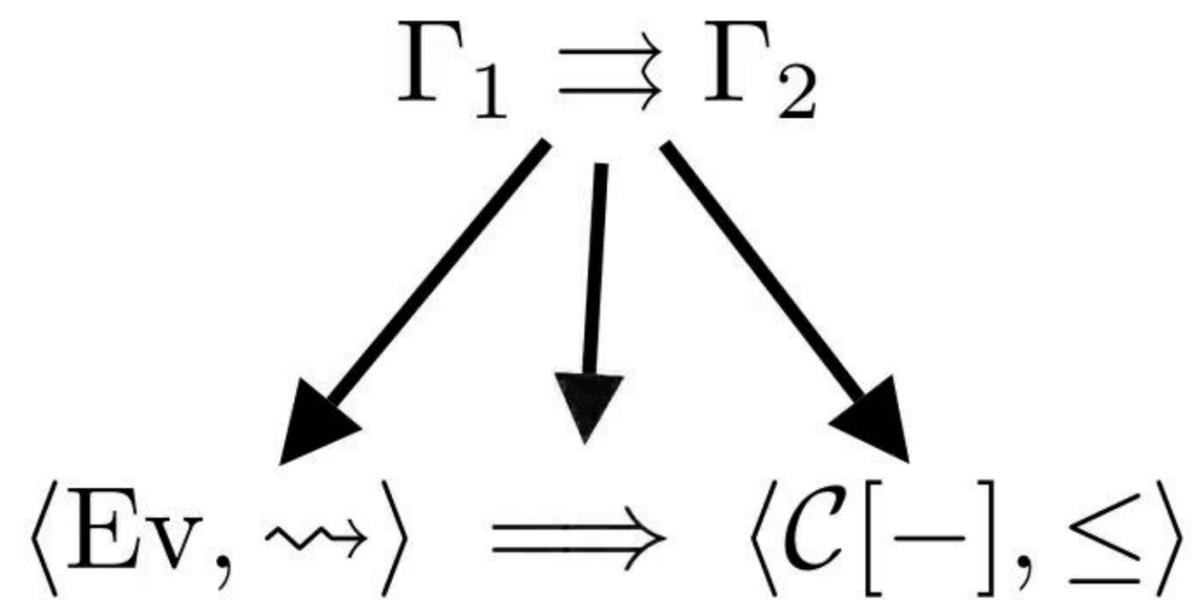
Inductively defined

Graphical syntax

Compositional semantics







Symmetric monoidal categories?
Separation logic?

Symmetric monoidal categories?

Separation logic?

Symmetric monoidal categories?

Separation logic?

Thank you!

<https://github.com/lsc-ucsc/csds>