Protocol Verification

A Brief Introduction to Model Checking and Temporal Logic

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SPLV 2024 @ Strathclyde

29.07.2024 - 02.08.2024

Motivation

Protocol Verification?

Protocols

Examples of protocols

- Distributed systems (e.g. paxos) 

Protocols

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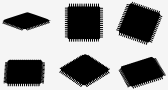
- Hardware (e.g. cache coherence) 

Protocols

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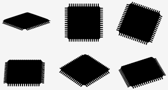
- Hardware (e.g. cache coherence) 
- Cryptographic protocols (e.g. TLS) 

Verification

Examples of properties

- Fairness 

Verification

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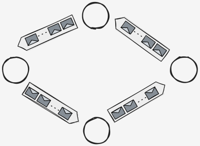
- Deadlock-freedom 

Verification

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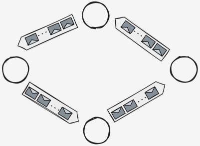
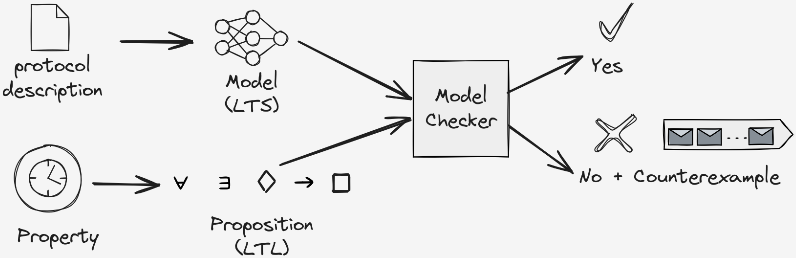
- Deadlock-freedom 
- Safety 

Protocol Verification

What this course is about



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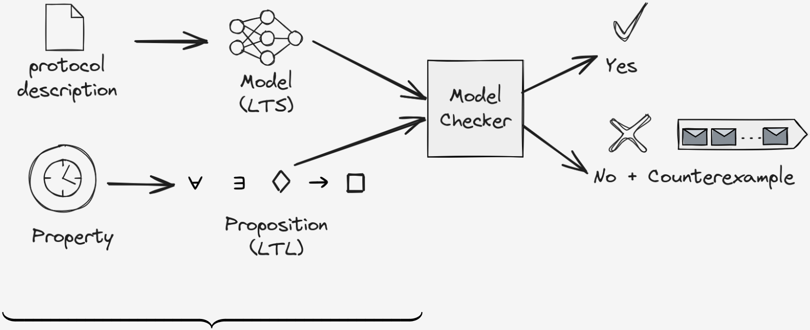

This is what we'll cover

Overview of the course

What you *will* (hopefully) know by the end

- Labeled transition systems (LTS)
- Modeling languages (promela)
- (Propositional) Linear Temporal Logic (LTL)
- Examples!

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What you will *not* (necessarily) know by the end

- Other logics (e.g. CTL*, μ calculus)
- How model checking works internally (decision procedures)

Modelling Protocols

Labeled Transition Systems

Definition (Labeled Transition Systems)

A labeled transition system is a tuple of the form $(S, \text{Act}, \rightarrow, S_0, \text{AP}, L)$, where S is a set of states, $S_0 \subseteq S$ a subset of initial states, Act is a set (of actions), $\rightarrow \subseteq \text{Act} \times S \times S$ is a (transition) relation, AP is a set (of atomic propositions) and $L : S \rightarrow \text{Pow}(\text{AP})$ is a (labeling) function.

Example: Traffic Light

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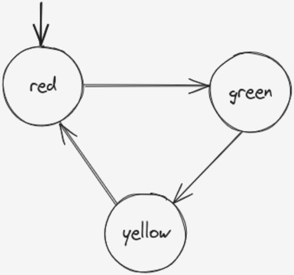

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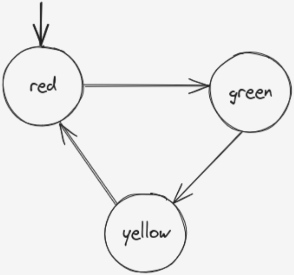
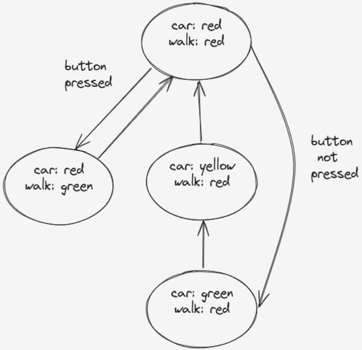


- $S = \{\text{red}, \text{green}, \text{yellow}\}, \; S_0 = \text{red}$
- $\text{Act} = \{*\}$
- $\rightarrow = \{(*, \text{red}, \text{green}), (*, \text{green}, \text{yellow}), (*, \text{yellow}, \text{red})\}$
- $\text{AP} = L = \emptyset$.

Two Traffic Lights



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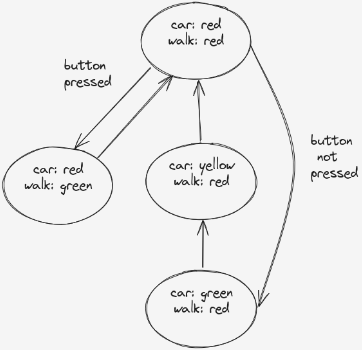

- $\text{Act} = \{\epsilon, \text{button pressed}, \text{no button pressed}\}$
- $\text{AP} = \{\text{Pedestrians can go}, \text{Cars can go}\}$
- $L = \text{cars: red, walk: green} \mapsto \{\text{Pedestrians can go}\}, \ldots$

Interleaving

Two traffic lights $\not\leftrightarrow$ One LTS

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Two traffic lights $\nleftrightarrow$ One LTS

Definition (Interleaving)

Let $TS_i = (S_i, \mathrm{Act}_i, \rightarrow_i, S_{0,i}, \mathrm{AP}_i, L_i), i = 1, 2$ be two transition systems. We define the transition system $TS_1 \interleave TS_2 :=$ $(S_1 \times S_2, \mathrm{Act}_1 \cup \mathrm{Act}_2, \rightarrow, S_{0,1} \times S_{0,2}, \mathrm{AP}_1 \cup \mathrm{AP}_2, L_1 \times L_2)$, where $L_1 \times L_2 : S_1 \times S_2 \rightarrow \mathrm{Pow}(\mathrm{AP}_1 \cup \mathrm{AP}_2)$ is defined as $(L_1 \times L_2)(s_1, s_2) = L_1(s_1) \cup L_2(s_2)$ and $\rightarrow$ is defined by

$$\frac{s_1 \rightarrow_1^{\alpha} s_1'}{(s_1, s_2) \rightarrow^{\alpha} (s_1', s_2)} \qquad \frac{s_2 \rightarrow_2^{\alpha} s_2'}{(s_1, s_2) \rightarrow^{\alpha} (s_1, s_2')} \ .$$

We call this construction the *interleaving* of TS_1 and TS_2.

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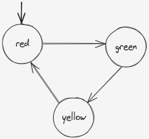
Note that this means the two TS are *independent*

Example: Intearleaving



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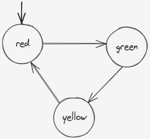

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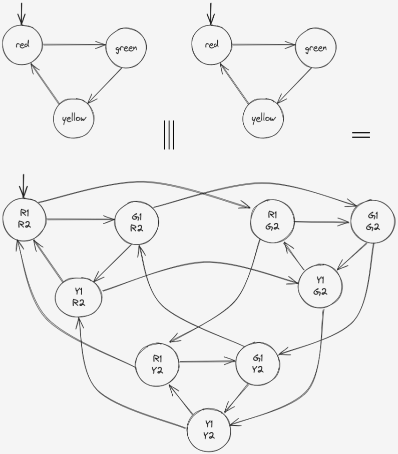


Parallel Composition

Definition (Handshake)

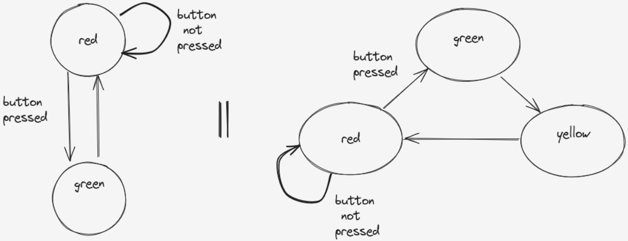
Let $TS_i = (S_i, \mathrm{Act}_i, \rightarrow_i, S_{0,i}, \mathrm{AP}_i, L_i), i = 1, 2$ be two transition systems and $H \subseteq \mathrm{Act}_1 \cap \mathrm{Act}_2$. We define the transition system $TS_1 \parallel_H TS_2 := (S_1 \times S_2, \mathrm{Act}_1 \cup \mathrm{Act}_2, \rightarrow$ $, S_{0,1} \times S_{0,2}, \mathrm{AP}_1 \cup \mathrm{AP}_2, L_1 \times L_2)$, where $\rightarrow$ is defined by:

$$\frac{s_1 \rightarrow_1^\alpha s_1' \quad \alpha \notin H}{(s_1, s_2) \rightarrow^\alpha (s_1', s_2)} \qquad \frac{s_2 \rightarrow_1^\alpha s_2' \quad \alpha \notin H}{(s_1, s_2) \rightarrow^\alpha (s_1, s_2')}$$

$$\frac{s_1 \rightarrow_1^\alpha s_1' \quad s_2 \rightarrow_2^\alpha s_2' \quad \alpha \in H}{(s_1, s_2) \rightarrow^\alpha (s_1', s_2')}$$

We call this the *parallel composition with handshake H*. When $H = \mathrm{Act}_1 \cap \mathrm{Act}_2$, we omit H.

Two Traffic Lights, revisited



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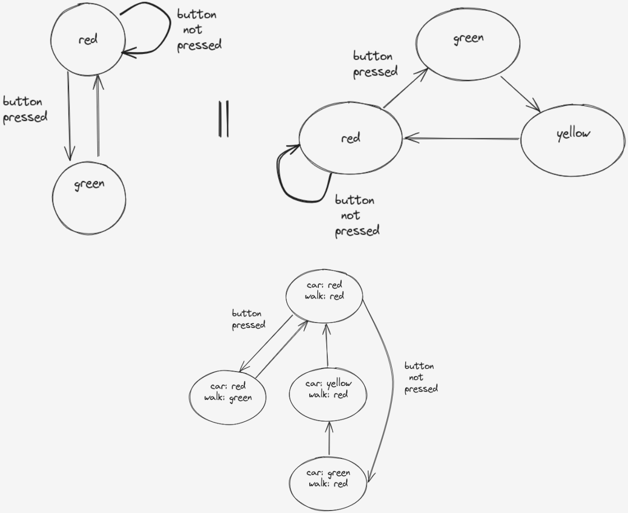
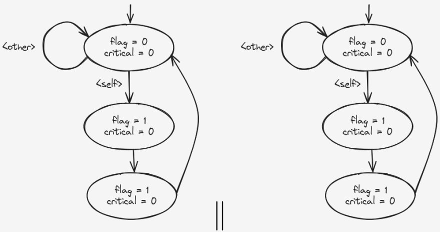


Concurrency: Message Passing



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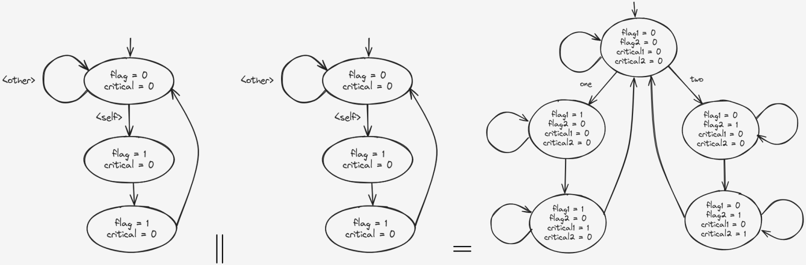

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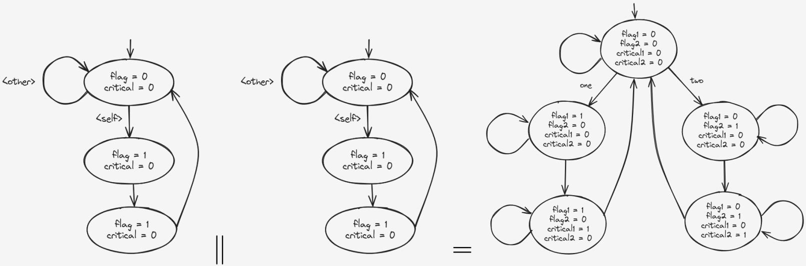
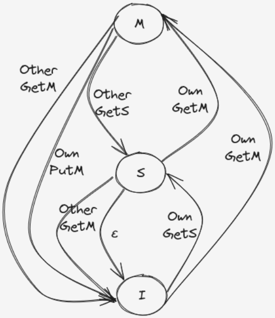


Assumption: atomicity of read-modify-writes here. Reasonable?

MSI Cache Coherency Protocol



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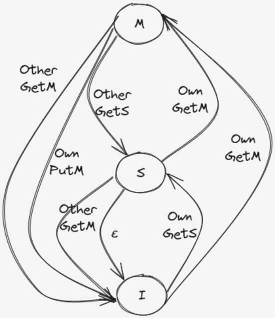

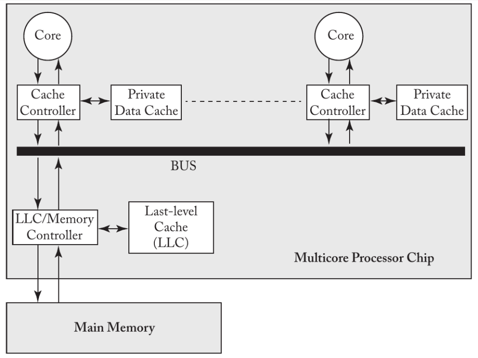

Source: Nagarajan, Vijay, et al. A primer on memory consistency and cache coherence. Springer Nature, 2020.

State Graph

- TS $\neq$ Graphs

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- Visualization (graphs): very useful!

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Definition (Predecessors/Successors)

Let $TS = (S, \text{Act}, \rightarrow, S_0, \text{AP}, L)$ be a transition system. For $s \in S, \alpha \in \text{Act}$, we define $\text{Post}(s, \alpha) := \{s' \in S \mid s \rightarrow^{\alpha} s'\}, \text{Post}(s) := \bigcup_{\alpha \in \text{Act}} \text{Post}(s, \alpha)$ as the successors of s, and similarly Pre for the predecessors.

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Definition (State Graph)

Let $TS = (S, \text{Act}, \to, S_0, \text{AP}, L)$ be a transition system. We call the directed graph $G(TS) = (S, E)$ the state graph of TS, where $E = \{s, s' \in S \times S \mid s \in S, s' \in \text{Post}(s)\}$

Path Fragments

Definition (Path fragments)

Let $TS = (S, \text{Act}, \rightarrow, S_0, \text{AP}, L)$ be a transition system. A sequence $\pi = \pi_0\pi_1\pi_2 \ldots \in (S)_{\mathbb{N}}$ is called a *path fragment* if $\pi_{i+1} \in \text{Post}(\pi_i) \forall i \in \mathbb{N}$. It is called *finite* if it is a finite sequence $(\pi_i)_{i=0}^{N}$instead.

For a path fragment π, we denote the i-th element by $\pi[i]$ and similarly the sub-sequence $(\pi_k)_{k=i}^{j}$ by $\pi[i..j]$

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Sequences of transitions = path framgents through the state graph

Paths

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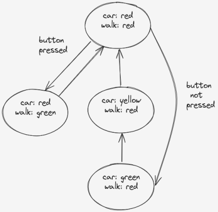
Definition (Path)

A path fragment π is called a *path* if it is initial and maximal.

Example: Paths in Traffic Light



A Typical Traffic Light in the UK?

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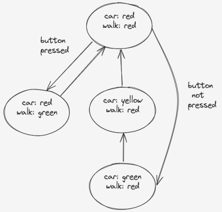

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Non-example

Finite vs Infinite Paths

finite path fragments can be extended to infinite ones, but...

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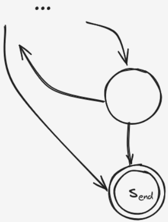

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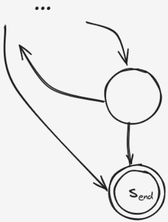
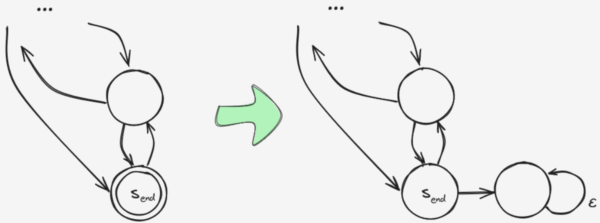


$$\text{Post}(s) = \varnothing$$

End States

Modeling end states with infinite paths



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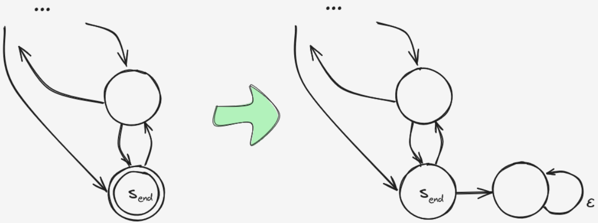

Assumption

For the rest of this course we assume no end states s with $\text{Post}(s) = \emptyset$.

Traces

- Paths $\triangleq$ sequences of states $\in S$

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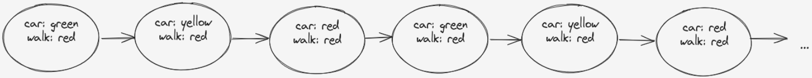
- Properties defined over AP, not S

Definition (Traces)

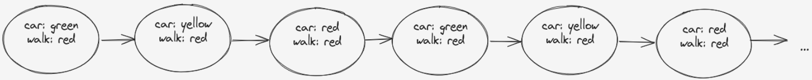
Let π be a path fragment. We define the *trace* of π as the sequence $L(\pi) \in (\mathbb{N} \to \mathrm{Pow}(\mathrm{AP}))$ as the sequence given by $(L(\pi))_i = L(\pi_i) \forall i \in \mathbb{N}$, and similarly for a finite path fragment. For $s \in S$ we define $\mathrm{Traces}(s)$ as the set of traces for path fragments starting at s, and $\mathrm{Traces}(TS) = \bigcup_{s \in S_0} \mathrm{Traces}(s)$.

Example: Traces



Corresponds to

Example: Traces



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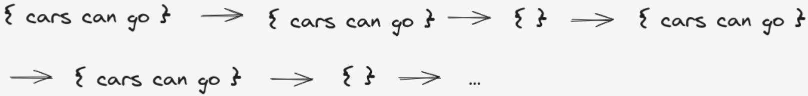


Equivalence of LTSs

- Many notions of equivalence.

Equivalence of LTSs

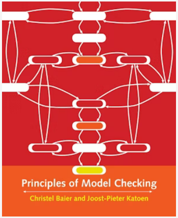
- Many notions of equivalence.
- Today: one

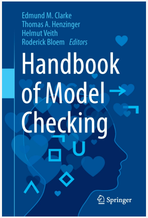
Definition (Trace Equivalence)

Let $TS_i, i = 1, 2$ be two tranisition systems with $\mathrm{AP}_1 = \mathrm{AP}_2$. We say TS_1 and TS_2 are *trace equivalent* if $\mathrm{Traces}(TS_1) = \mathrm{Traces}(TS_2)$.

References

Main references for this course:



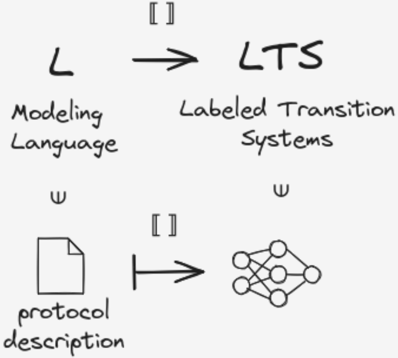


- Baier, Christel, and Joost-Pieter Katoen. Principles of model checking. MIT press, 2008.
- Clarke, Edmund M., et al., eds. Handbook of model checking. Vol. 10. Cham: Springer, 2018.

Modeling Languages: An Introduction to Promela

Modelling Languages

Core Idea



Promela

- Spin: mature model checker (>30 years of development)

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- Spin: mature model checker (>30 years of development)
- Promela = **Pro**tocol/cess **me**ta **la**nguage
- C-inspired syntax

Hello Promela

```
init{
    int num = 11 * 23 * 8;
    printf("Hello SPLV %d\n", num);
}
```

Hello Promela

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init{
    int num = 11 * 23 * 8;
    printf("Hello SPLV %d\n", num);
}
```

```
→  splv24 git:(master) ✗ spin promela-examples/hello.pml
      Hello SPLV 2024
1 process created
```

Do Blocks

```
#define N 100

proctype counter(int i){
    do // repeats indefinitely
    :: (i < N) -> i = i + 1  // guarded increase
    :: (i >= N) -> break // break do loop
    od
    end: skip // declare a (valid) end state
}

init{
    run counter(0)
}
```

Promela: Traffic Lights

```
mtype = {red, green, yellow}
mtype car = red;
mtype walk = red;

active proctype TrafficLight(){
    do
    :: (walk == red && car == red) -> car = green
    :: (walk == red && car == red) -> walk = green
    :: (car == red && walk == green) -> walk = red
    :: car == green -> car = yellow
    :: car == yellow -> car = red
    od
}
```

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    :: car == yellow -> car = red
    od
}
```
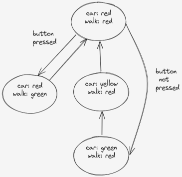
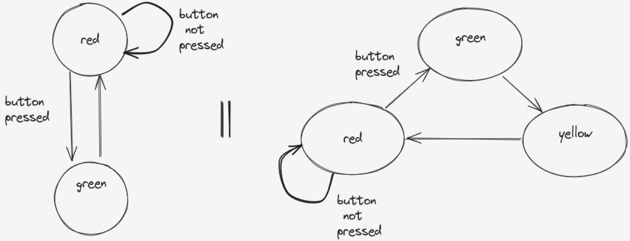



Composition

Recall:



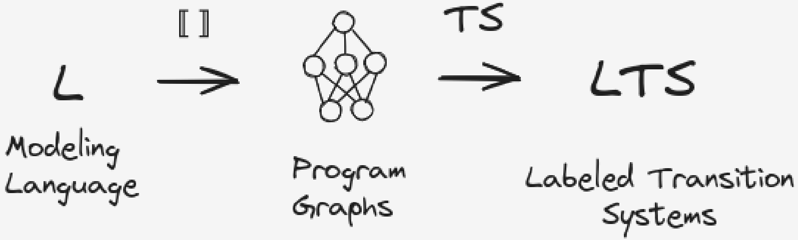
Communication (Channels)

```
mtype = {red, green, yellow}
mtype car = red;
mtype walk = red;

// Channel of size 0 = synchronous communication
chan press = [0] of {bool};

active proctype PedestrianButton(){
    do
    :: press!true // send `true`
    :: press!false // send `false`
    od
}
```

Communication (Channels) contd.

```
active proctype TrafficLight(){
    bool button_pressed = false;
    do
    :: (walk == red && car == red) ->
       press?button_pressed; //receive pressed
       if
       :: button_pressed -> walk = green
       :: !button_pressed -> car = green
       fi
    :: (car == red && walk == green) -> walk = red
    :: car == green -> car = yellow
    :: car == yellow -> car = red
    od
}
```

Program Graphs



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- Transition relation from this:

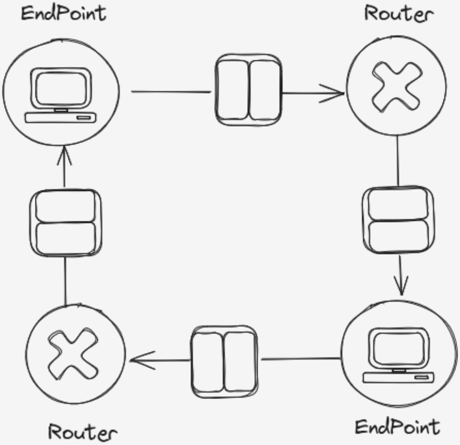
$$\frac{l \hookrightarrow^{g,\alpha} l' \quad \eta \models g}{(l,\eta) \rightarrow^{\alpha} (l', ([\![\alpha]\!])(\eta))}$$

Example: MP Concurrency

```
bool flag[2]; //flag for entering critical section
byte num_crit; //how many processes in critical section

active [2] proctype user()        // two processes
{
do
::
       flag[_pid] = 1;
       flag[1 - _pid] == 0 ->
           num_crit = num_crit + 1; // enter
           num_crit = num_crit - 1; // exit
       flag[_pid] = 0;
od
}
```

Example: Buffers



Example: Buffers in Promela

```
mtype = {request, response, nil}

proctype Router(chan buffer_from, buffer_to){
    mtype msg = nil;
    do /* a router just keeps forwarding messages */
    :: buffer_from?msg  -> buffer_to!msg
    od
}
```

Example: Buffers in Promela (ctd.)

```
proctype EndPoint(chan buffer_from, buffer_to){
    mtype msg = nil;
    do
    :: atomic{ (msg == nil) && buffer_from?[msg]
    -> buffer_from?msg}
    :: atomic{ (msg == request)
    -> buffer_to!response; msg = nil }
    :: atomic{ (msg == response) ->  msg = nil }
    :: buffer_to!request
    od
}
```

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- Extend actions Act with set of communication actions Comm
- Comm: Actions $c!v$ and $c?x$ to send value v on channel c and receive into variable x.
- Multiple program graphs: composition ($\|$) with matching actions built from $c?v/c!v$ pairs.

Modelling Properties

Models

What is a model?

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Model Theory 101

- Structures, e.g. groups, rings, fields, *labeled transition systems*

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- Formulas in a given logic, e.g. $a = b$, $\exists c, a * c = 1$, $\Box(\neg p)$
- Models $A \models \phi$, i.e. the formula ϕ holds in the structure A

(Modal) Logic

- Propositional logic $(P, Q, \ldots, \vee, \wedge, \neg)$

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- First-order logic $(P, Q, \ldots, \vee, \wedge, \neg, \exists, \forall)$
- Modal logic $(\ldots, \Box, \Diamond)$
  - $\Box \approx$ necessity
  - $\Diamond \approx$ possibility

Linear-Time Properties

Definition (LT Property)

Let $TS = (S, \text{Act}, \rightarrow, S_0, \text{AP}, L)$ be a transition system. A *linear property* of TS is a set of traces, i.e. sequences $P \subseteq \text{AP}^{\mathbb{N}}$ over atocmic propositions AP.

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Idea: these are the admissible traces in TS

Definition (Satisfying an LT Property)

Let $TS = (S, \mathrm{Act}, \rightarrow, S_0, \mathrm{AP}, L)$ be a transition system and let P be a linear time property. We say that TS satisfies P, in symbols, $TS \models P$, iff $\mathrm{Traces}(TS) \subseteq P$.

Linear Temporal Logic (intro)

- Propositional logic + modal operators
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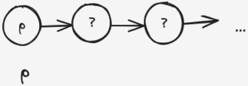
- Note: propositional logic (and LTL) has no quantifiers $\forall, \exists$ (!)

Intuition of LTL Operators



- $p \in AP$
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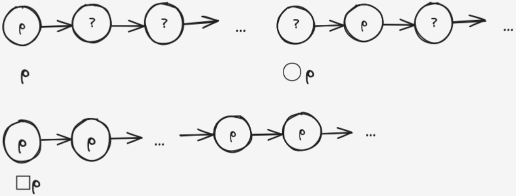

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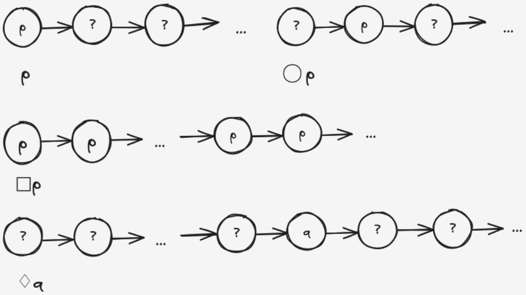

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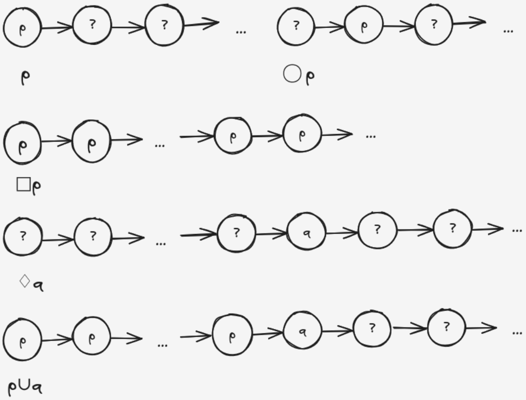
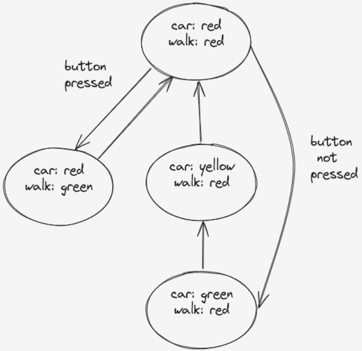


Example: Safety



“Cars and Pederstrians can never go at the same time ”

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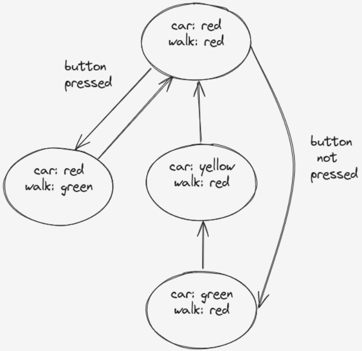

"Cars and Pederstrians can never go at the same time " $\triangleq$
$\Box\neg(\text{cars can go} \wedge \text{pedestrians can go})$

LTL, Formally (Syntax)

Definition (Syntax of LTL)

Let AP be a set (of atomic propositions). Then, an LTL formula over AP is a word in the language defined by the grammar:

$$\varphi ::= \text{true} \mid a \mid \varphi_1 \wedge \varphi_2 \mid \neg\varphi \mid \bigcirc\varphi \mid \varphi_1 \cup \varphi_2$$

We call the set of such formulae LTL_{AP}. When AP is clear from context, we also say φ is an LTL formula (and omit AP).

LTL, Formally (Semantics, I)

Definition (The "Models" Relation)

We define $\models$ as the minimial relation over traces and LTL formulae $\models \subseteq (\mathbb{N} \to \mathsf{Pow}(\mathsf{AP})) \times \mathsf{LTL}_{\mathsf{AP}}$, such that:

$$
\begin{array}{ll}
A \models \mathsf{true} & \\
A \models a \in \mathsf{AP} & \text{iff } a \in A_0 \\
A \models \varphi_1 \wedge \varphi_2 & \text{iff } A \models \varphi_1 \text{ and } A \models \varphi_2 \\
A \models \neg\varphi & \text{iff } A \not\models \varphi \\
A \models \bigcirc\varphi & \text{iff } A[1\ldots] = A_1 A_2 \ldots \models \varphi \\
A \models \varphi_1 \cup \varphi_2 & \text{iff } \exists j,\ A[j\ldots] \models \varphi_2 \text{ and } \forall i < j,\ \sigma[i\ldots] \models \varphi_1
\end{array}
$$

LTL, Formally (Semantics, II)

Definition (Semantics of LTL)

Let φ be an LTL formula over AP. We define
$\mathsf{Words}(\varphi) := \{\pi \in \mathsf{Pow}(\mathsf{AP})^{\mathbb{N}} \mid \pi \models \varphi\}$.

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Definition

We say the transition system *TS satisfies* φ (in symbols, $TS \models \varphi$), if $\text{Traces}(TS) \subseteq \text{Words}(\varphi)$.

Temporal Modalities

Definition ($\Diamond$ Operator)

For an LTL formula φ, we define the operator $\Diamond$ as

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Definition ($\Box$ Operator)

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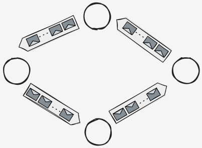
$$\Box\varphi := \neg\Diamond\neg\varphi$$

Deadlocks

Recall: 

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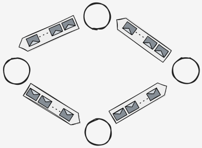
Recall: 

Temporal logic?

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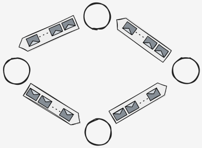
Recall: 

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Recall: we assumed no finite states

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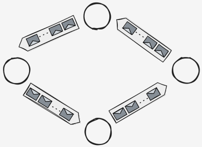
Recall: 

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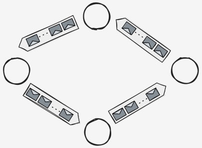
Recall: 

Temporal logic?

Recall: we assumed no finite states

- transformation is a deadlock check
- no deadlock $\triangleq \Box\neg$capture-state

Invariants

Invariant (property does not change) $\triangleq \Box P$

Examples:

- mutual exclusion: never two process in critical section
  $\Box(\mathit{crit} < 2)$
- cars and pedestrians don't go at the same time
  $\Box(\neg \text{cars can go} \lor \neg \text{pederstrians can go})$

Safety

Other safety properties: bad prefix

- Yellow should warn of red coming:
  $\Box(\neg(\text{yellow} \vee \text{red}) \rightarrow \bigcirc \neg \text{red})$

Definition

An LT property P over AP is called a *safety property*, if for all traces $\pi \in \text{Pow}\,(\text{AP})^{\mathbb{N}}$ there exists a finite prefix $\hat{\pi} \sqsubset \pi$ such that extensions of that prefix are disjoint from P, i.e.
$\{\pi' \in \text{Pow}\,(\text{AP})^{\mathbb{N}} \mid \hat{\pi} \sqsubset \pi'\} \cap P = \varnothing$

Fairness

"Everybody gets their turn"

- Unconditional $\Box\Diamond P$ ("Everybody gets their turn infinitely often")
- Strong $\Box\Diamond P \rightarrow \Box\Diamond Q$ ("Everybody who asks infinitely often, goes infinitely often")
- Weak $\Diamond\Box P \rightarrow \Box\Diamond Q$("Everybody who is waiting from some point on, gets their turn infinitely often")
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(Nondeterminism): Condition or constraint?

Liveness Properties

More generally, liveness are things of the type "good thing happen infinitely often"

- Traffic light's let people through: $\Box\Diamond\text{green}$
- Mutex lets processes do their work: $\Box((\Diamond\text{crit}_1) \wedge (\Diamond\text{crit}_2))$

Definition (Liveness — Alpern and Schneider)

An LT property P over AP is called a *liveness property*, if every finite word can be extended to a trace in the property P, i.e. for all $\hat{\pi} \in \text{Pow}(\text{AP})^*$ there exsits a $\pi \in P$ such that $\hat{\pi} \sqsubset \pi$.

Decomposition

- Safety properties: constrain finite behavior
- Liveness properties: constrain infinite behavior

Theorem

Let P be a linear time property P over AP, i.e. $P \subseteq \mathrm{Pow}\,(AP)^{\mathbb{N}}$. Then there exist a liveness property P_{live} and a safety property P_{safe} over AP, such that $P = P_{live} \cap P_{safe}$.

Decomposition Theorem

Proof.

(Sketch) The metric

$$d : \mathsf{Pow}(AP)^{\mathbb{N}} \times \mathsf{Pow}(AP)^{\mathbb{N}} \to \mathbb{R}_{\geqslant 0},$$
$$(\pi, \sigma) \mapsto \begin{cases} 0, & \text{if } \sigma = \pi \\ \frac{1}{|gcp(\sigma,\pi)|}, & \text{otherwise} \end{cases},$$

where $gcp(\sigma, \pi)$ denotes the greatest common prefix of σ and π, makes $\mathsf{Pow}(AP)^{\mathbb{N}}$ a metric space. Safety properties are the closed sets of the induced topology. We have

$$P = \underbrace{\bar{P}}_{:=P_{\mathsf{safe}}} \cap \underbrace{P \cup (\mathsf{Pow}(AP)^{\mathbb{N}}) \setminus \bar{P})}_{:=P_{\mathsf{live}}}$$

□

Model Checking with Spin

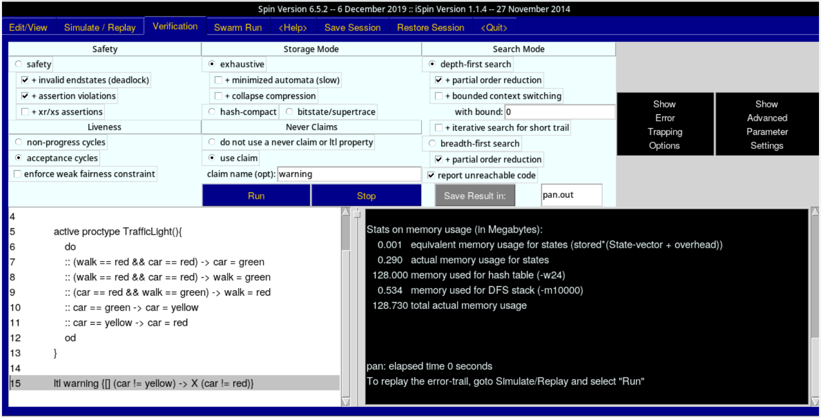
- Deadlocks: nothing additional! (`end` label)

Model Checking with Spin

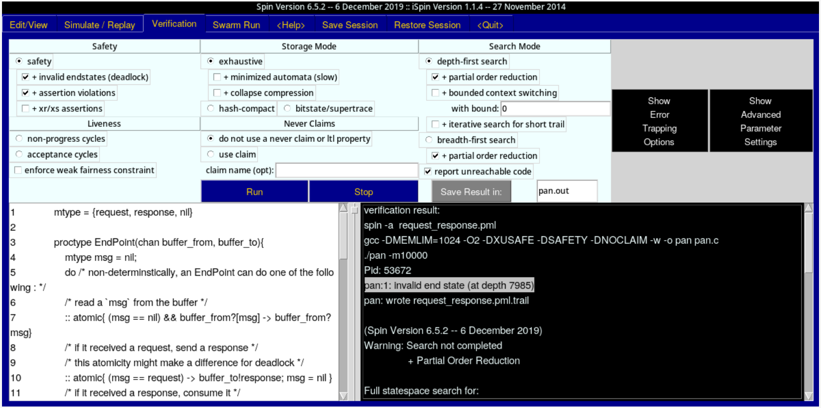
- Deadlocks: nothing additional! (`end` label)
- LTL Formulae: `never` claims

```
-f LTL Translate the LTL formula LTL into a never claim.
       This option reads a formula in LTL syntax from the second argument and  translates
       it into Promela syntax (a never claim, which is Promela's equivalent of a Bchi Au-
       tomaton).  The LTL operators are written: []  (always),  <>  (eventually),  and  U
       (strong  until).   There is no X (next) operator, to secure compatibility with the
       partial order reduction rules that are applied during  the  verification  process.
       If  the  formula contains spaces, it should be quoted to form a single argument to
       the SPIN command.
       This option has largely been replaced with the support for inline specification of
       ltl formula, in Spin version 6.0.
```

Example: Safety in Traffic Light



Example: Deadlock in Request-



A Word on Complexity

- Invariant checking (BFS) is linear in state space, formula, transitions (still large spaces!).
- General LTL model checking is PSPACE hard

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- General LTL model checking is PSPACE hard
- Mitigations:
  - Partial order reduction
  - Symmetry reduction
  - Abstraction (gradual refinements)
  - Symbolic model checking
  - ...