

# Proving Behavioural Apartness

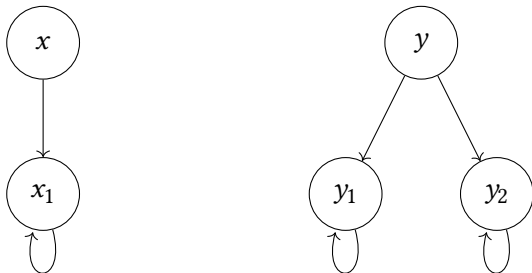
SPLV 2024

Ruben Turkenburg

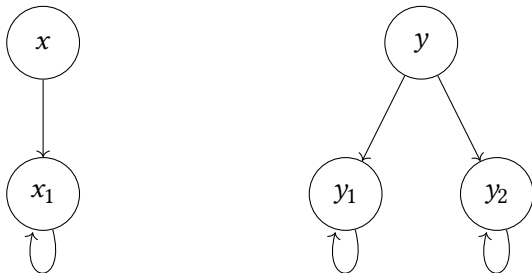
based on joint work with  
Harsh Beohar, Clemens Kupke and Jurriaan Rot

2024-07-29

## State-Transition Systems

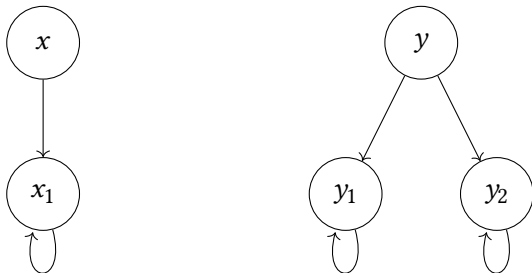


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- In this example:  $\gamma : \{x, x_1, y, y_1, y_2\} \rightarrow \mathcal{P}_f(\{x, x_1, y, y_1, y_2\})$

## Equivalence or Distinguishability?

- Equivalences defined parametrically in the transition type of the coalgebra:
  - Behavioural Equivalence
  - Coalgebraic Bisimilarity
  - Codensity Bisimilarity

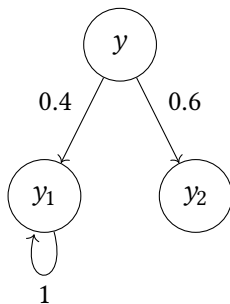
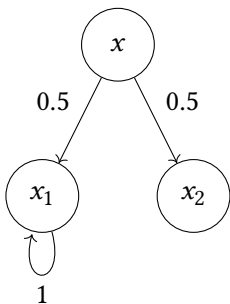
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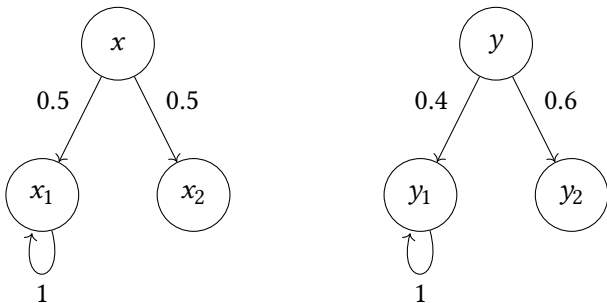
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- How do we *prove* states to be equivalent?
- Easier: proof system for apartness (Geuvers & Jacobs 2021)

## Systems with probabilities





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$x_1 \# x_2$

$y_1 \# y_2$

$x_2 \# y_1$

$x_1 \# y_2$

$$\frac{\mu_x[x_1]_{\bar{R}} = 0.5 \neq 0.4 = \mu_y[x_1]_{\bar{R}}}{x \# y}$$

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  - Relation to (quantitative) modal logics

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- From proof to distinguishing formulas?